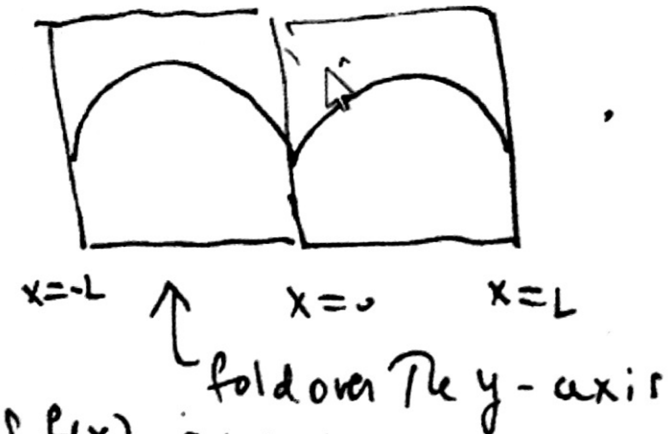
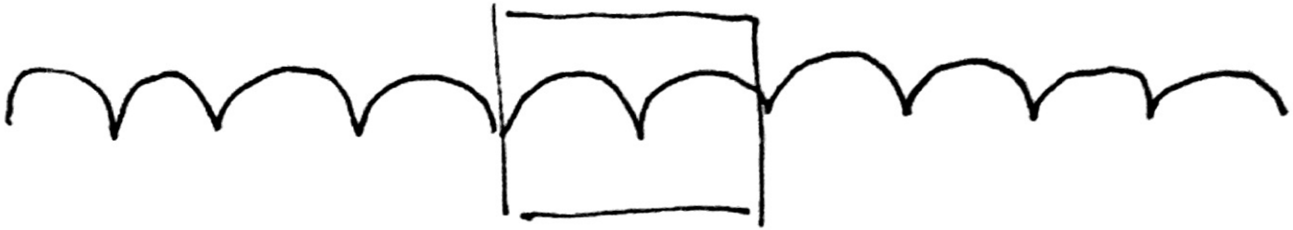


$$\text{Base function} = \begin{cases} f(x) & 0<x<L \\ f(-x) & -L<x<0 \end{cases} = \text{even function}$$



Base function
is even

assume $f(x)$ is piecewise Smooth
Can be discont.

(1) Integration of Fourier Series term-by-Term
ANS: Yes

(2) Differentiation of Fourier Series Term by Term
ANS: May be

$$f(x) = \begin{cases} 1 & 0<x<1 \\ 2+x & 1<x<2 \\ x^2 & 2<x<3 \end{cases}$$

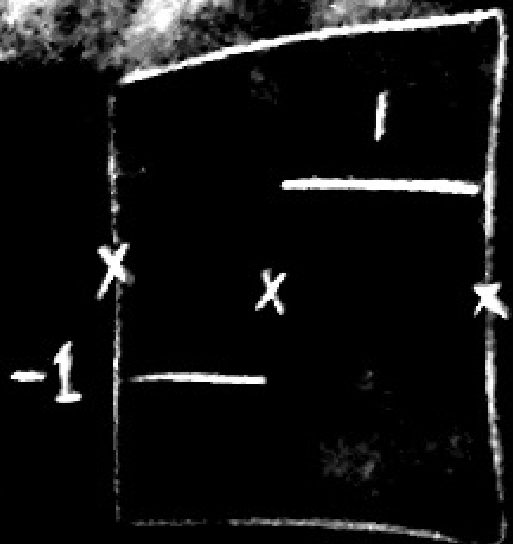
Each has a Cont. Der.

(1.1) Theorem f piecewise smooth $\Longrightarrow$

$\int_{-L}^{x} f(x)\,dx =$ Term-by-Term integration of the Fourier series $\int_{-L}^{x}$

$$\int_a^b = \int_a^x + \int_x^b$$

$$= \int_a^x - \int_b^x$$



$$F(x) = \frac{\pi}{4} \sum_{k=0}^{\infty} \int_{-L}^{x} \frac{\sin\left((2k+1)\frac{\pi x}{L}\right)}{2k+1}\, dx = \int_{-L}^{x} f(x)\, dx$$

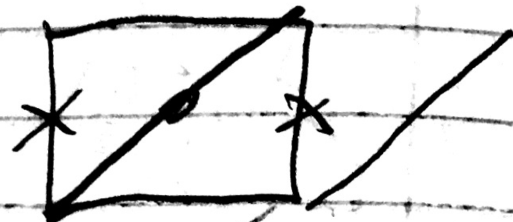
~~Except at jumps~~ all x

$|x|$

$$f(x)=\begin{cases} 0 & x<-1 \\ |x| & -1<x<1 \\ 1+x & x>1 \end{cases}$$

① Theorem

$$x = \sum_{n=1}^{\infty} \frac{2L(-1)^{n+1}}{n\pi} \sin\left(\frac{n\pi x}{L}\right)$$



Bag MT

valid for $0 \leq x < L$

$$\frac{d}{dx} x = \sum_{n=1}^{\infty} \frac{2L(-1)^{n+1}}{n\pi} \sin\left(\frac{n\pi x}{L}\right) \frac{d}{dx}$$

$$1 = \sum_{n=1}^{\infty} 2(-1)^{n+1} \cos\left(\frac{n\pi x}{L}\right)$$

does not converge n^{th} term test

(1) Theorem f piecewise smooth

- $f(x)$ is cont (no jumps)
- $f(-L) = f(L)$

$$\Rightarrow \frac{d}{dx}\begin{pmatrix}\text{Fourier Series} \\ \text{Term-by-Term}\end{pmatrix} = \frac{d}{dx} f(x)$$

(2) Theorem

(3) Theorem

(1) Theorem f piecewise smooth

$f(x)$ is cont (no jumps)

$f(-L)=f(L)$

$$\Rightarrow \frac{d}{dx}\begin{pmatrix}\text{Fourier Series}\\ \text{Term-by-Term}\end{pmatrix} = \frac{d}{dx}f(x)$$

(2) Theorem f is P. S.

$f(x)$ cont (no jumps)

$f'(x)$ is piecewise smooth (no corners)

$\Rightarrow \frac{d}{dx}f(x) =$ Term-by-Term $\frac{d}{dx}$ of Fourier Cosine Series

(3) Theorem

(1) Theorem f piecewise smooth

$f(x)$ is cont (no jumps)

$f(-L) = f(L)$

$\Longrightarrow \frac{d}{dx}\left(\begin{array}{c}\text{Fourier Series}\\ \text{Term-by-Term}\end{array}\right) = \frac{d}{dx} f(x)$

(2) Theorem f is P. S.

$f(x)$ cont (no jumps)

$f'(x)$ is piecewise smooth (no corners)

$\Rightarrow \frac{d}{dx} f(x) =$ Term-by-Term $\frac{d}{dx}$ of Fourier Cosine Series

(3) Theorem

$\longleftarrow$ assume

$\longleftarrow$ assume

$f(0)=0,\ f(L)=0$

$\Longrightarrow$ True for Sine Series

FFT

Uses Complex Fourier Series

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{-ik\pi x/L} = a_0 + \sum_{1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$c_k = \frac{a_k + ib_k}{2} \quad k>0, \quad c_0 = a_0, \quad c_{-k} = \frac{a_k - ib_k}{2}, \quad k>0$$

Orthogonal set on $[-L, L]$

$$\left\{ e^{in\pi x/L} \right\}_{n=-\infty}^{n=\infty}$$

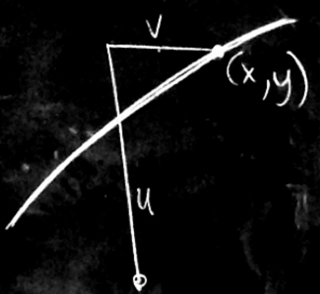
$$\int_{-L}^{L} \bar{h}_1 h_2 \, dx = 0 \quad \text{if } h_1 \neq h_2$$

$-$ means complex conjugate

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\theta = \frac{n\pi x}{L}$$

Wave Equation



assume

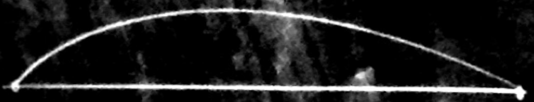
(1) $y = u(x,t)$

(2) Ignore V

(3) Tension = constant = T_0

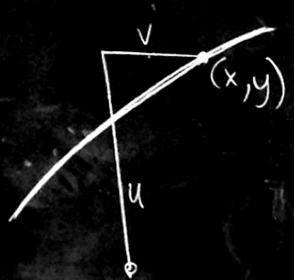
(4) perfectly flexible

(5) $\rho_0(x)$ = mass density = ρ_0



Wave Equation

assume



(1) $y = u(x,t)$

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(5) $\rho_0(x)$ = mass density = ρ_0

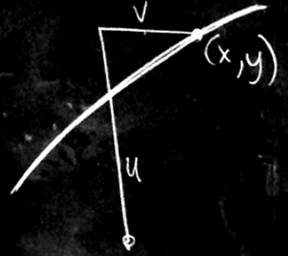
$$\rho_0(x)\, u_{tt} = T_0\, u_{xx} + Q(x,t)\,\rho_0(x)$$

$Q = -g$ usually

$$\rho_0\, u_{tt} = T_0\, u_{xx}$$

$Q = 0$

Wave Equation



assume

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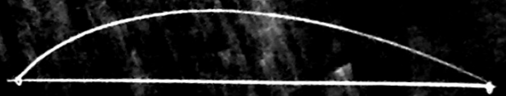


$$\rho_0(x)\, u_{tt} = T_0\, u_{xx} + Q(x,t)\,\rho_0(x)$$

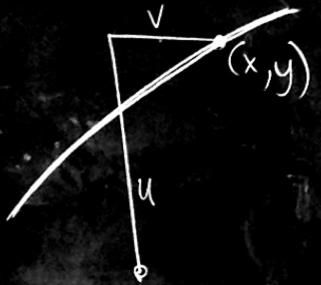
$Q = -g$ usually

$$\rho_0\, u_{tt} = T_0\, u_{xx}, \quad Q = 0$$

$$u_{tt} = c^2 u_{xx}$$

$\frac{T_0}{\rho_0} \equiv c^2$, c = Wave Speed

Wave Equation



assume

(1) $y = u(x,t)$

(2) Ignore V

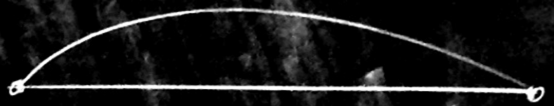
(3) Tension = constant $= T_0$

(4) perfectly flexible

(5) $\rho_0(x) =$ mass density $= \rho_0$

$$\boxed{u_{tt} = c^2 u_{xx}}$$

+ shape
+ Speed



$$\rho_0(x)\, u_{tt} = T_0\, u_{xx} + Q(x,t)\rho_0(x)$$

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$\frac{T_0}{\rho_0} \equiv c^2$, $c =$ Wave Speed