


$$\sin(a+b) = \sin(a) \cos(b) + \sin(b) \cos(a)$$

$$a = \frac{n\pi u}{L}, \quad b = \frac{n\pi L/2}{L} = \frac{n\pi}{2} = \frac{2k\pi}{2} = k\pi$$

$$\sin\left(\frac{n\pi u}{L} + \frac{n\pi L/2}{L}\right) = (-1)^k \sin\left(\frac{n\pi u}{L}\right)$$

Trick:

$$b_n = \frac{2}{L} (-1)^k \int_{-L/2}^{L/2} f(u + \frac{L}{2}) \sin\left(\frac{n\pi u}{L}\right) du$$


$u=0$

$f(u + \frac{L}{2})$
Even function

$$b_n = \frac{2}{L} (-1)^k \int_{-L/2}^{L/2} (\text{even})(\text{odd}) du$$