

KEY Example

Is $\int_0^L \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{3\pi x}{L}\right) dx = 0$?

No. Should be $-L < x < L$.

Theorem

Fourier series of $f =$ Fourier cosine series of f_1
+ Fourier sine series of f_2

$$f_1(x) = \frac{f(x) + f(-x)}{2}, \quad f_2(x) = \frac{f(x) - f(-x)}{2}$$