

Case $\lambda < 0$

$$\lambda = -\omega^2$$

$$X = c_1 e^{\omega x} + c_2 e^{-\omega x}$$

Not bounded
No Sol

Case $\lambda = 0$

$$X = c_1 + c_2 x$$

$$c_2 = 0$$

$$\boxed{X = 1, \lambda = 0}$$

Case $\lambda > 0$

$$\lambda = \omega^2$$

$$\boxed{X = c_1 \cos(\omega x) + c_2 \sin(\omega x)}$$

$$\lambda = \omega^2 > 0$$

Complex form $X(x)$

$$X = c_1 \cos(\omega x) + c_2 \sin(\omega x)$$

$$= c_1 \frac{e^{i\omega x} + e^{-i\omega x}}{2} + c_2 \frac{e^{i\omega x} - e^{-i\omega x}}{2i}$$

$$= d_1 e^{i\omega x} + d_2 e^{-i\omega x}$$

$$X = \text{constant} * e^{i\omega x} \quad -\infty < \omega < \infty$$

Eigenpairs

$$\lambda = \omega^2 \quad 0 \leq \omega < \infty$$

$$(\lambda, \cos \omega x), (\lambda, \sin \omega x)$$

$$\lambda = \omega^2, \quad \omega = \omega \text{ or } -\omega$$

$$(\lambda, e^{i\omega x})$$