

Char eq: $Y^2 + \lambda = 0$

3 Cases

(1) Real roots $\lambda < 0$

(2) double root $\lambda = 0$

(3) Complex roots $\lambda > 0$

$$\text{Discr} = b^2 - 4ac$$

$$= 0 - 4 \cdot 1 \cdot \lambda$$

$$= -4\lambda$$

$$Y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

$$Y = (C_1 + C_2 x) e^{r_1 x}$$

$$Y = C_1 \cos(\sqrt{\lambda} x) + C_2 \sin(\sqrt{\lambda} x)$$

Monotonic

Case 3 $Y = 0$ at $x = 0, x = L$

Means

$$x = 0: 0 = C_1 \overset{1}{\cancel{\cos(0)}} + C_2 \overset{0}{\cancel{\sin(0)}}$$

$$x = L: 0 = C_2 \sin(\sqrt{\lambda} L)$$

$$\sqrt{\lambda} L = n\pi$$