

**Math 2250 Extra Credit Maple Lab 6, Mechanical Oscillations.
S2014**

Due date: See the internet due dates. Maple lab 6 has four problems: L6-1, L6-2, L6-3. Please answer the questions A, B, C, ... associated with each problem.

Problem scores for the Spring 2014 version of maple lab 6, Mechanical Vibrations:

- _____ L6-1. [100] Under-Damped Free Vibration.
- _____ L6-2. [100] Undamped Forced Vibration.
- _____ L6-3. [100] Practical Resonance.

L6-1. Under-Damped Free Vibration

Free Vibrations. Consider the problem of free linear vibrations of a damped spring-mass system

$$\begin{aligned} mx'' + cx' + kx &= 0, \\ x(0) &= 0, \quad x'(0) = 1, \\ m &= 4, \quad c = 3. \end{aligned}$$

Symbols m , c and k are non-negative constants, representing the mass, viscous damping constant and Hooke's constant, respectively. The under-damped case is studied here, $c^2 < 4km$, as explained in section 5.4 the Edwards-Penney textbook *Differential Equations and Linear Algebra*, 3/E. Inserting $m = 4$, $c = 3$ gives the requirement $9 < 16k$ on Hooke's constant k .

Problem L6-1.

- A. Select and display a positive Hooke's constant k so that the solution $x(t)$ is under-damped. Choose the specific value of k so that the graphic in part B below displays well. Check that $x(t) = 0$ for infinitely many $t \geq 0$ (the solution oscillates). Display the exact solution $x(t)$ obtained by Maple methods as in the example below.
- B. Plot the exact symbolic solution $x(t)$ on a suitable t -interval. Check the graphic against Figure 5.4.9 in Edwards-Penney.
- C. Estimate from the graph the decimal value of the pseudo-period. Display the graphical estimate and also the exact pseudo-period $2\pi/w$, where w is the natural frequency of the trigonometric term in the solution $x(t)$ found above in item L6.1-A.

```
# EXAMPLE(Wrong parameters! Change it!)
# Use semicolons to see what you have done.
# Define the differential equation
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)+4*x(t)=0:
# Solve the characteristic equation.
solve(3*r^2+1.5*r+4=0,r);
# Define the initial conditions
ic:=x(0)=0,D(x)(0)=1:
# Symbolically solve for x(t)
p:=dsolve({de,ic},x(t),method=laplace):
# Capture the dsolve symbolic solution as a function X(t)
X:=unapply(rhs(p),t):
# Plot the solution
plot(X(t),t=0..5);
```

Maple tip: Click with the mouse on the graphic to print the cursor location (left upper corner of the maple window). The coordinates printed are of the form (x, y) . From this coordinate information, a subtraction estimates the period.

L6-2. Undamped Forced Vibration

Forced Linear Vibrations. Consider the undamped ($c = 0$) forced vibration problem for a spring-mass system:

$$\begin{aligned} mx'' + kx &= 5 \cos(wt), \\ x(0) &= 0, \quad x'(0) = 0, \\ m &= 5, \quad k = 3.5 \end{aligned}$$

Symbol w is a positive constant, the input natural frequency. Symbols m, k are respectively mass and Hooke's constant.

- A. Divide the differential equation by $m = 5$ to obtain $x'' + w_0^2 x = \cos(wt)$ where $w_0^2 = k/m = 3.5/5$ defines the natural angular frequency $w_0 = \sqrt{35/50} = 0.8366600265$. Choose the input natural frequency w to be 3 times larger than the natural angular frequency w_0 . Solve for $x(t)$ using Maple's `dsolve()`.

- B. Because $w = 3w_0$, then $w \neq w_0$. The solution $x(t)$ is the sum of two functions, one of period $2\pi/w$ and the other of period $2\pi/w_0$ (phenomenon of Beats). Graph the slowly-varying envelope curves and the rapidly-varying solution curve $x(t)$ on a suitable interval. See Figure 5.6.3 in Edwards-Penney.
- C. Suggest a value for the forcing frequency w so that the vibration exhibits resonance. Show the resonant behavior in a graphic. Check against Figure 5.6.4 in Edwards-Penney.

L6-3. Practical Resonance

Consider the damped forced vibration problem

$$\begin{aligned} mx'' + cx' + kx &= 5 \cos(wt), \\ x(0) &= 0, \quad x'(0) = 0, \\ m &= 4, \quad k = 41. \end{aligned}$$

Symbol w is a positive constant, the input natural frequency. Symbols m, c, k are respectively mass, viscous damping constant and Hooke's constant.

- A. Consider the damping constants $c = 2$, $c = 1$ and $c = 1/2$. Compute the amplitude function $C(w)$ [section 5.6] for these three equations, then plot for $w = 0$ to $w = 20$ the three amplitude graphs on a single set of axes. Compare against Figure 5.6.9 in Edwards-Penney (it has one curve, yours has 3 curves).
- B. For each case $c = 2$, $c = 1$, $c = 1/2$, print the values w^* , C^* where $C^* = C(w^*) = \max\{C(w) : 0 \leq w \leq 20\}$. The three data pairs should show that C^* becomes larger as c tends to zero.

Maple Hint: Use Maple's mouse interface on the graphic of Part A. Specifically, click on a possible maximum (horizontal tangent) in the graph to display the values w^* , C^* on the screen. Look around the screen to see where maple printed the x, y -coordinates! Copy the values into your maple worksheet report.

```
#EXAMPLE(Beware! Wrong values!)
F:=15: m:=1: k:=25:
c:='c': w:='w':
C:=(w,c)->F/sqrt((k-m*w*w)^2+(c*w)^2):
plot({C(w,4),C(w,3),C(w,2)},w=0..15,color=black);
```