Name ____________________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 10
S2014

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper **Extra Credit**. Label each solved problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc10.3-20. (Inverse transform)

Solve for $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{1}{s^4 - 8s^2 + 16}$. Use partial fractions in the details.

Problem Xc10.3-24. (Inverse transform)

Solve for $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{s}{s^4 + 4a^4}$, showing the details that give the answer $f(t) = \dfrac{1}{2a^2}\sinh at \sin at$

Problem Xc10.4-12. (Inverse transform, convolution)

Solve for $f(t))$ in the relation $\mathcal{L}(f(t)) = \dfrac{1}{s(s^2 + 4s + 5)}$. Instead of the convolution theorem, use partial fractions for the details. If you can see how, then check the answer with the convolution theorem.

Problem Xc10.4-26. (Inverse transform techniques)

Use the s-differentiation theorem in the details of solving for $f(t)$ in the relation $\mathcal{L}(f(t)) = \arctan \dfrac{3}{s+2}$. You will need to apply the theorem $\lim_{s\to\infty} \mathcal{L}(f(t)) = 0$.

Problem Xc10.4-40. (Series methods for transforms)

Expand in a series, using Taylor series formulas, the function $f(t) = \dfrac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$. Then find $\mathcal{L}(f(t))$ as a series by Laplace transform of each series term, separately. Finally, re-constitute the series in variable s into elementary functions, namely $e^{-1/s}$ divided by $\sqrt{s}$.

Problem Xc10.5-6. (Second shifting theorem, Heaviside step)

Find the function $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{se^{-s}}{s^2 + \pi^2}$.

Problem Xc10.5-14. (Transforms of piecewise functions)

Let $f(t) = \begin{cases} \cos \pi t & 0 \le t \le 2, \\ 0 & t > 2. \end{cases}$ Find $\mathcal{L}(f(t))$. Details should expand $f(t)$ as a linear combination of Heaviside step functions.

Problem Xc10.5-26. (Sawtooth wave)

Let $f(t+a) = f(t)$ and $f(t) = t$ on $0 \le t \le a$. Then f is a-periodic and has a Laplace transform obtained from the periodic function formula. Show the details in the derivation to obtain the answer $\mathcal{L}(f(t)) = \dfrac{1}{as^2} - \dfrac{e^{-as}}{s(1 - e^{-as})}$.

Problem Xc10.5-28. (Modified sawtooth wave)

Let $f(t+2a) = f(t)$ and $f(t) = t$ on $0 \le t \le a$, $f(t) = 0$ on $a < t \le 2a$. Then f is $2a$-periodic and has a Laplace transform obtained from the periodic function formula. Derive a formula for $\mathcal{L}(f(t))$. The answer to this problem can be found in Edwards-Penney, section 10.5.

Problem Xc-EPbvp-7.6-8. (Impulsive DE)

Solve by Laplace methods $x'' + 2x' + x = \delta(t) - 2\delta(t-1)$, $x(0) = 1$, $x'(0) = 1$. Check the answer in `maple` using `dsolve({de,ic},x(t),method=laplace)`.

Problem Xc-EPbvp-7.6-18. **(Switching circuit)**

A passive LC-circuit has battery 6 volts and model equation $i'' + 100i = 6\delta(t) - 6\delta(t-1)$, $i(0) = 1$, $i'(0) = 1$. The switch is closed at time $t = 0$ and opened again at $t = 1$. Solve the equation by Laplace methods and report the number of full cycles observed before the steady-state $i = 0$ is reached (to two decimal places). Check the answer in `maple` using `dsolve({de,ic},i(t),method=laplace)`.

End of extra credit problems chapter 10.