

Math 2250 Extra Credit Problems Chapter 10X
Challenging Laplace Applications S2014

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper **Extra Credit**. Label each solved problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc10X.1. (Inverse transform)

Solve for $f(t)$, given $\mathcal{L}(f(t)) = e^{-2s} \frac{s}{(s+1)(s^2+1)}$.

Problem Xc10X.2. (Inverse transform)

Solve for $f(t)$, given $\mathcal{L}(f(t)) = \frac{d}{ds} \left(e^{-2s} \frac{s+1}{s^2(s^2+1)} \right)$.

Problem Xc10X.3. (Inverse transform)

Solve for $f(t)$, given $\mathcal{L}(f(t)) = \frac{s+4}{s^2(s^2+2s+2)} + \frac{e^{-2s}}{s(1-e^{-2s})}$. Hint: Look on the inside cover of Edwards-Penney.

Problem Xc10X.4. (Resolvent Equation and e^{At})

Leverrier and Faddeeva derived the following recursions for the coefficients in the expansion

$$(sI - A)^{-1} = \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{\det(sI - A)} A_k.$$

$$\det(sI - A) = s^n - \sum_{k=0}^{n-1} c_k s^k, \quad A_0 = I, \quad A_k = A_{k-1}A - c_{n-k}I.$$

Then

$$e^{At} = \mathcal{L}^{-1}((sI - A)^{-1}) = \sum_{k=0}^{n-1} \mathcal{L}^{-1} \left(\frac{s^{n-k-1}}{\det(sI - A)} \right) A_k.$$

- (a) Write out A_0, A_1 for a 2×2 matrix A , given $\det(sI - A) = s^2 - c_1s - c_0$.
- (b) Write out A_0, A_1, A_2 for a 3×3 matrix A , given $\det(sI - A) = s^3 - c_2s^2 - c_1s - c_0$.
- (c) Apply the Leverrier-Faddeeva formulas to find the matrix exponential e^{At} for the special case

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (d) Check the answer for e^{At} in a computer algebra system.

Problem Xc10X.5. (Laplace transform)

Find $\mathcal{L}(f(t))$, given $f(t) = \frac{\sinh(t)}{t}$.

Problem Xc10X.6. (Laplace transform)

Find $\mathcal{L}(f(t))$, given $f(t) = \frac{d}{dt}(\sinh(t) \cosh(t))$.

Problem Xc10X.7. (Laplace's method)

Solve by Laplace's method the initial value problem $tx''(t) - 2x'(t) + tx(t) = 0$, $x(0) = 1$, $x'(0) = 0$. Check the answer in maple.

Problem Xc10X.8. (Laplace's method)

Solve by Laplace's method the initial value problem $tx''(t) + 2x'(t) + x(t) = \delta(t) + \delta(t-1) + \delta(t-2)$, $x(0) = 0$, $x'(0) = 1$. Check the answer in maple using `dsolve({de,ic},x(t),method=laplace)`.

Problem Xc10X.9. (Backward table)

Solve for $f(t)$, given $F(s) = \mathcal{L}(f(t))$.

$$F(s) = \frac{5s^4 + 16s^3 + 2560s^2 + 1600s + 327680}{(s^2 + 100)(s^2 + 256)^2}.$$

Problem Xc10X.10. (Backward table)

Solve for $f(t)$, given $F(s) = \mathcal{L}(f(t))$.

$$F(s) = \frac{5s^4 - 24s^3 + 80s^2 - 32s + 256}{(s-2)^3(s^2+16)^2}.$$

End of extra credit problems chapter 10X.