Sample Quiz2 Problem 1.

Suppose a cup of hot chocolate has an initial temperature of $185°F$ when freshly poured and the desired drinking temperature is $160°F$. After 50 seconds in a room at $68°F$, the temperature has cooled to $181°F$. **Newton's Law of Cooling** applies to model the temperature $u(t)$ of the chocolate by the initial value problem

$$\frac{du}{dt} = -h\left(u(t) - 68\right), \quad u(0) = 185,$$

where $h > 0$ is the cooling constant, to be determined from supplied information.

1. Find an equation for the temperature $u(t)$ at any time t.

2. Find the Newton cooling constant h.

3. Determine the time required for the chocolate to cool to $160°F$.

References. Edwards-Penney section 1.5. Serway and Vuille, *College Physics 9/E*, Brooks-Cole (2011), ISBN-10: 0840062060. An answer check might use *The Coffee Cooling Problem*, a Wolfram Demonstration Project contributed by S.M. Binder, which can be found at http://demonstrations.wolfram.com/TheCoffeeCoolingProblem/.

Credits. Created by Rebecca Terry, January 2014.

Sample Quiz2 Problem 2. Logistic growth $F(x) = rx(1 - x/K)$ can be used to describe the annual natural growth of a fish stock. Symbol $x(t)$ is the stock biomass in number of fish at the start of month t. The intrinsic growth rate is symbol r. The environmental carrying capacity in stock biomass terms is symbol K.

1. Assume a fish pond has carrying capacity $K = 780500$ and that 80% of the the fish survive to maturity. We'll assume 6 months to maturity and $r = 0.8$. Write in detail the no-harvesting model $x'(t) = F(x(t))$ and find the equilibrium values.

2. Assume constant harvesting H to give the model $x'(t) = F(x(t)) - H$. Use the quadratic formula from algebra to find the equilibrium points as a function of symbol $H \geq 0$. Then verify the following results.

   If $H = 156100$, then there are two states: extinction for $x(0) < 390250$ and limiting population 390250 otherwise.

   If $H > 156100$, then the extinction state is the only possibility.

   If $H < 156100$, then there are two equilibria. The larger equilibrium population size is stable and the smaller is unstable. These numbers imply sustainable harvest for certain population sizes, but not all.

3. Assume a constant harvest rate H. Create two graphics of the population $x(t)$ over 36 months. The first uses a harvesting size H to show sustainable harvest. The second uses a different size H to show non-sustainable harvest. Handwritten plots are expected, or a computer plot, if you know how.

References. Edwards-Penney sections 2.1, 2.2. Course documents: Logistic Equation, Stability, Fish Farming and a logistic investigation in Malaysia by M.F. Laham 2012.