Quiz 4

Quiz4 Problem 1. The velocity of a crossbow bolt launched upward from the ground was determined from a video and a speed gun to complete the following table.

Time t in seconds	Velocity $v(t)$ in ft/sec	Location
0.000	60	Ground
1.7	0	Maximum
3.5	-52	Near Ground Impact

(a) The bolt velocity can be approximated by a quadratic polynomial

$$v(t) = at^2 + bt + c$$

which reproduces the table data. Find three equations for the coefficients a, b, c. Then solve for the coefficients.

(b) Assume a linear drag model $v' = -32 - \rho v$. Substitute the polynomial answer of (a) into this differential equation, then substitute $t = 0$ and solve for $\rho \approx 0.11$.

(c) Solve the model $w' = -32 - \rho w$, $w(0) = 60$ with $\rho = 0.11$.

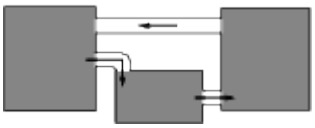
(d) The error between $v(t)$ and $w(t)$ can be measured. Is the drag coefficient value $\rho = 0.11$ reasonable?

References. Edwards-Penney sections 2.3, 3.1, 3.2. Course documents on Linear algebraic equations and Newton kinematics.

Quiz4 Problem 2. Consider the system of differential equations

$$\begin{aligned} x_1' &= -\tfrac{1}{5}x_1 + \tfrac{1}{7}x_3, \\ x_2' &= \tfrac{1}{5}x_1 - \tfrac{1}{3}x_2, \\ x_3' &= \tfrac{1}{3}x_2 - \tfrac{1}{7}x_3, \end{aligned}$$

for the amounts x_1, x_2, x_3 of salt in recirculating brine tanks, as in the figure:



Recirculating Brine Tanks A, B, C

The volumes are $50, 30, 70$ for A, B, C, respectively.

The steady-state salt amounts in the three tanks are found by formally setting $x_1' = x_2' = x_3' = 0$ and then solving for the symbols x_1, x_2, x_3.

(a) Solve the corresponding linear system of algebraic equations for answers x_1, x_2, x_3.

(b) The total amount of salt is uniformly distributed in the tanks in ratio $5 : 3 : 7$. Explain this mathematically from the answer in (a).

References. Edwards-Penney sections 3.1, 3.2, 7.3 Figure 5. Course documents on Linear algebraic equations and Systems and Brine Tanks.