

**Quiz3 Problem 1.** A graphic called a **phase diagram** displays the behavior of all solutions of  $u' = F(u)$ . A **phase line diagram** is an abbreviation for a direction field on the vertical axis ( $u$ -axis). It consists of equilibrium points and signs of  $F(u)$  between equilibria. A phase diagram can be created solely from a phase line diagram, using just three drawing rules:

1. Solutions don't cross.
2. Equilibrium solutions are horizontal lines  $u = c$ . All other solutions are increasing or decreasing.
3. A solution curve can be moved rigidly left or right to create another solution curve.

Use these tools on the equation  $u' = (u-1)(u-2)^2(u+2)$  to make a phase line diagram, and then make a phase diagram with at least 8 threaded solutions. Label the equilibria as stable, unstable, funnel, spout, node.

**References.** Edwards-Penney section 2.2. Course document on Stability,

**Quiz3 Problem 2.** An autonomous differential equation  $\frac{dy}{dx} = F(x)$  with initial condition  $y(0) = y_0$  has a formal solution

$$y(x) = y_0 + \int_0^x F(u) du.$$

The integral may not be solvable by calculus methods. In this case, the integral is evaluated numerically to compute  $y(x)$  or to plot a graphic. There are three basic numerical methods that apply, the rectangular rule (RECT), the trapezoidal rule (TRAP) and Simpson's rule (SIMP).

Apply the three methods for  $F(x) = \cos(x^2)$  and  $y_0 = 0$  using step size  $h = 0.2$  from  $x = 0$  to  $x = 1$ . Then fill in the blanks in the following table. Use technology if it saves time. Lastly, compare the four data sets in a plot, using technology.

$x$ - values	0.0	0.2	0.4	0.6	0.8	1.0
$y$ - to 10 digits	0.0	0.1999680024	0.3989772129	0.5922705167	0.7678475376	0.9045242379
$y$ - RECT values	0.0	0.2	0.3998400213	0.5972854780	<input type="text"/>	0.9448839943
$y$ - TRAP values	0.0	0.1999200107	0.3985627497	<input type="text"/>	0.7646744186	0.8989142250
$y$ - SIMP values	0.0	0.1999666703	0.3989746144	0.5922670741	0.7678445414	<input type="text"/>

**References.** Edwards-Penney Sections 2.4, 2.5, 2.6, because methods Euler, Modified Euler and RK4 reduce to RECT, TRAP, SIMP methods when  $f(x, y)$  is independent of  $y$ , i.e., an equation  $y' = F(x)$ . Course document on numerical solution of  $y' = F(x)$  RECT, TRAP, SIMP methods. Wolfram Alpha at <http://www.wolframalpha.com/> can do the RECT rule and graphics with input string

```
integrate cos(x^2) using left endpoint method with interval width 0.2 from
x=0 to x=1
```