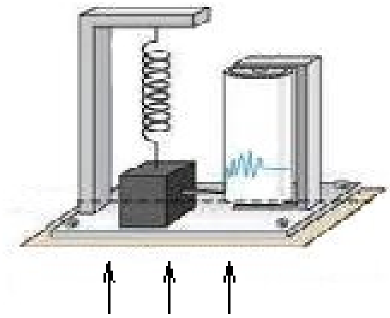
Quiz 11

Sample Quiz 11, Problem 1. Vertical Motion Seismoscope

The 1875 **horizontal motion seismoscope** of F. Cecchi (1822-1887) reacted to an earthquake. It started a clock, and then it started motion of a recording surface, which ran at a speed of 1 cm per second for 20 seconds. The clock provided the observer with the earthquake hit time.



A Simplistic Vertical Motion Seismoscope

The apparently stationary heavy mass on a spring writes with the attached stylus onto a rotating drum, as the ground moves up. The generated trace is $x(t)$.

The motion of the heavy mass m in the figure can be modeled initially by a forced spring-mass system with damping. The initial model has the form

$$mx'' + cx' + kx = f(t)$$

where $f(t)$ is the vertical ground force due to the earthquake. In terms of the vertical ground motion $u(t)$, we write via Newton's second law the force equation $f(t) = -mu''(t)$ (compare to falling body $-mg$). The final model for the motion of the mass is then

$$(1) \qquad \begin{cases} x''(t) + 2\beta\Omega_0 x'(t) + \Omega_0^2 x(t) = -u''(t), \\ \dfrac{c}{m} = 2\beta\Omega_0, \quad \dfrac{k}{m} = \Omega_0^2, \\ x(t) = \text{center of mass position measured from equilibrium}, \\ u(t) = \text{vertical ground motion due to the earthquake.} \end{cases}$$

Terms **seismoscope, seismograph, seismometer** refer to the device in the figure. Some observations:

Slow ground movement means $x' \approx 0$ and $x'' \approx 0$, then (1) implies $\Omega_0^2 x(t) = -u''(t)$. The seismometer records ground acceleration.

Fast ground movement means $x \approx 0$ and $x' \approx 0$, then (1) implies $x''(t) = -u''(t)$. The seismometer records ground displacement.

A **release test** begins by starting a vibration with u identically zero. Two successive maxima $(t_1, x_1), (t_2, x_2)$ are recorded. This experiment determines constants β, Ω_0.

The objective of (1) is to determine $u(t)$, by knowing $x(t)$ from the seismograph.

The Problem.

Assume the seismograph trace can be modeled at time $t = 0$ (a time after the earthquake struck) by $x(t) = 10\cos(3t)$. Assume a release test determined $2\beta\Omega_0 = 16$ and $\Omega_0^2 = 80$. Explain how to find a formula for the ground motion $u(t)$, then provide details for the answer $u(t) = \frac{710}{9}\cos(3t) - \frac{160}{3}\sin(3t)$ (assume integration constants are zero).

Quiz 11, Problem 2. Laplace Theory

Laplace theory implements the *method of quadrature* for higher order differential equations, linear systems of differential equations, and certain partial differential equations.

Laplace's method **solves differential equations**.

The Problem. Solve by table methods or Laplace's method.

(a) Forward table. Find $\mathcal{L}(f(t))$ for $f(t) = 3(t+1)^2 e^{2t} + 2e^t \sin(3t)$.

(b) Backward table. Find $f(t)$ for

$$\mathcal{L}(f(t)) = \frac{4s}{s^2+4} + \frac{s-1}{s^2-2s+5}.$$

(c) Solve the initial value problem $x''(t) + 2x'(t) + 5x(t) = e^t$, $x(0) = 0$, $x'(0) = 1$.