Name KEY 1a, ver1

Scores
4
5

Differential Equations and Linear Algebra 2250

Midterm Exam 1a
Version 1, 23 Feb 2012

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

4. (Linear Equations)

(a) [50%] Solve the linear model. Show all integrating factor steps.

$$\begin{cases} 4x'(t) &= -128 + \dfrac{16}{2t+1}x(t), \\ x(0) &= 16 \end{cases}$$

(b) [20%] Solve the homogeneous equation $\dfrac{dy}{dx} - (2x+1)y = 0$.

(c) [30%] Solve $17\dfrac{dy}{dx} + 34y = \dfrac{5}{2}$ using the superposition principle $y = y_h + y_p$. Expected are answers for y_h and y_p

(a) $x' - \frac{4}{2t+1}x = \frac{-128}{4} \Rightarrow \frac{(xw)'}{w} = -32 \Rightarrow xw = c - 32\int w\,dt$

$w = e^{\int p\,dt} = e^{-2\ln|2t+1| + c_1}$ Choose $w = (2t+1)^{-2}$, Then

$xw = c - 32 \cdot \frac{(2t+1)^{-1}}{-1} \cdot \frac{1}{2} \Rightarrow xw = c + 16(2t+1)^{-1} \Rightarrow$

$x(2t+1)^{-2} = c + 16(2t+1)^{-1} \Rightarrow x(t) = c(2t+1)^2 + 32t + 16$

Then $x(0) = 16 \Leftrightarrow 16 = c + 16$ implies $c = 0$ and $\boxed{x(t) = 32t + 16}$

Ans check: maple dsolve

(b) The answer is: $y = \dfrac{\text{constant}}{\text{integrating factor}} = \dfrac{c}{e^{-\int(2x+1)dx}} = \boxed{\dfrac{c}{e^{-x^2-x}}}$

(c) Select y_p: $0 + 34y = \frac{5}{2}$ or $\boxed{y_p = \frac{5}{(34)(2)}}$

Solve for $y_h = \dfrac{c}{\text{integr. factor}}$

$y_h = \dfrac{c}{e^{\int p\,dx}} = \dfrac{c}{e^{2x}} \Rightarrow \boxed{y_h = ce^{-2x}}$

$\boxed{y = y_h + y_p}$

Use this page to start your solution Attach extra pages as needed.

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5. (Stability)

(a) [50%] Draw a phase line diagram for the differential equation

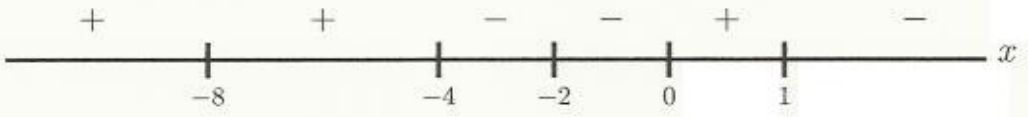
$$\frac{dx}{dt} = \cosh(x)\,(2-|2x-4|)^3\,(2+x)(x^2-4)(1-x^2)^3.$$

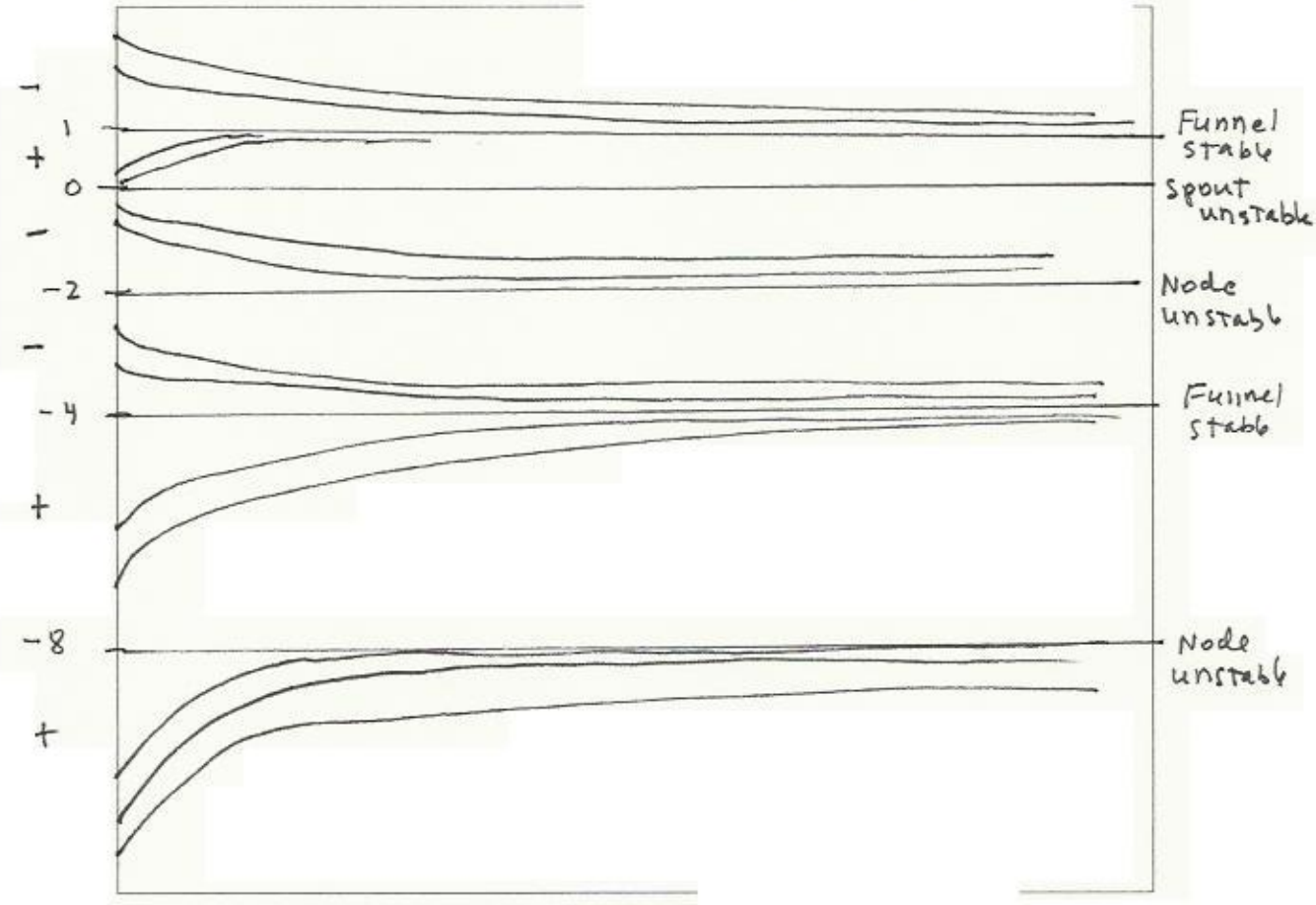
Expected in the phase line diagram are equilibrium points and signs of dx/dt. Definition. $\cosh(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

$\cosh(x) > 0$ for all x; $2-|2x-4| = 0 \iff 2x-4 = \pm 2 \iff x=3$ or $x=1$

Other roots are $x=-2, -2, 2, 1, -1$. Zero is not a root. There are 5 roots.

(b) [50%] Assume an autonomous equation $x'(t) = f(x(t))$. Draw a phase diagram with at least 12 threaded curves, using the phase line diagram given below. Add these labels as appropriate: **funnel**, **spout**, **node** [neither spout nor funnel], **stable**, **unstable**.





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