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$y' = ky$

Growth-Decay Equation

Solution $y = y_0 e^{kx}$

$y_0 =$ an arbitrary constant
$= y(0)$

$\frac{du}{dt} = -h(u - u_0)$

Newton's Cooling Equation

Solution $u = u_0 + A_0 e^{-ht}$

Obtained by changing $y = u - u_0$ to get $y' = -hy$, Then apply the recipe above.

$\frac{dP}{dt} = (a - bP)P$

Verhulst Logistic Equation

Solution

$$p(t) = \frac{a P_0}{b P_0 + (a - b P_0) e^{-at}}$$

where $P_0 = P(0) =$ initial population.

obtained by changing

$$y = \frac{P}{a - bP}$$

to get $y' = ay$, Then apply the recipe above.

Tyson Black

100

1.2 - #1

JWB 113

P 16

(1) Find a function $y = f(x)$ satisfying the given differential equation and the prescribed initial condition.

$\frac{dy}{dx} = 2x + 1$; $y(0) = 3$ — Given

$y(x) = \int (2x+1)\,dx$ — integration of both sides of equ.

$y(x) = x^2 + x + C$

use of $y(0) = 3$

$y(0) = 0 + 0 + C$

$0 + 0 + C = 3$

$C = 3$

Substitute 3 for C

$y(x) = x^2 + x + 3$

Check:

Back of Book

1.2 — #2

PROBLEM 2: $\boxed{\text{pg. 17 \#2}}$ Jennifer Lahti

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Find a function $y = f(x)$ which satisfies the given differential equation $\frac{dy}{dx} = (x-2)^2$ and initial condition $y(2)=1$.

$y'(x) = (x-2)^2$ — given initial equation

$y'(x)\,dx = (x-2)^2\,dx$ — apply the method of quadrature

$\int y'(x)\,dx = \int (x-2)^2\,dx$

$y(x) = \frac{(x-2)^3}{3} + c$

$1 = \frac{(2-2)^3}{3} + c$ — use $y(2)=1$

$c = 1$

$y(x) = \frac{(x-2)^3}{3} + 1$ — candidate solution

Check:

$LHS = y'(x)$

$= \left[\frac{(x-2)^3}{3} + 1\right]'$

$= (x-2)^2 + 0$

$= RHS$ — checks with initial differential equation

$LHS = y(2)$

$= \left[\frac{(x-2)^3}{3} + 1\right]\Big|_{x=2}$

$= 0 + 1$

$= RHS$ — checks with initial condition $y(2)=1$.

$\boxed{1.2-5}$

PROBLEM 3 $\boxed{\text{pg 17 \#5}}$ Jennifer L...

Find a function $y=f(x)$ which satisfies the given differential equation $\frac{dy}{dx}=\frac{1}{\sqrt{x+2}}$ and the initial condition $y(2)=-1$

$y'(x)=\frac{1}{\sqrt{x+2}}$ same procedure as problems #1 & 2

$\int y'(x)\,dx = \int \frac{dx}{\sqrt{x+2}}$

$y(x)=\int u^{-1/2}\,du$ apply "u" substitution

$y(x)=2u^{1/2}+c$

$y(x)=2\sqrt{x+2}+c$

$-1=2\sqrt{2+2}+c$

$c=-5$

$y(x)=2\sqrt{x+2}-5$

Check: candidate solution agrees with solution given in book.

Apply the method of quadrature to solve

$$\begin{cases} y' = 2x+1 \\ y(0) = 3 \end{cases}$$

$y' = 2x+1$	Given DE
$\int y'dx = \int (2x+1)dx$	Integrate across both sides on x.
$y = x^2 + x + c$	Fund. Thm. of calculus applied; c = constant
$3 = 0^2 + 0 + c$	use $y=3$ at $x=0$
$c = 3$	
$y = x^2 + x + 3$	Candidate solution found.
<u>Check:</u>	
$LHS = y'$ $= (x^2+x+3)'$ $= 2x+1$ $= RHS$	LHS = left hand side of $y' = 2x+1$, RHS = right hand side.
	DE verified
$y(0) = 0^2 + 0 + 3$ $= 3$	Initial condition $y(0)=3$ is verified.
$y = x^2 + x + 3$	Solution.

1.2 — #2

Find a function $y = y(x)$ which satisfies the differential equation $\frac{dy}{dx} = (x-2)^2$ and initial condition $y(2) = 1$.

$y'(x) = (x-2)^2$ Given

$y'(x)\,dx = (x-2)^2\,dx$ Apply the method of quadratures

$\int y'(x)\,dx = \int (x-2)^2\,dx$

$y(x) = \frac{(x-2)^3}{3} + C$

$1 = \frac{(2-2)^3}{3} + C$ Use $y(2)=1$ to find C

$C = 1$

$y(x) = \frac{(x-2)^3}{3} + 1$ Candidate solution

Check:

$LHS = y'(x)$ Left side of DE

$= \left[\frac{(x-2)^3}{3} + 1\right]'$

$= (x-2)^2 + 0$

$= RHS$ DE verified

$LHS = y(2)$ Left side of IC

$= \left[\frac{(x-2)^3}{3} + 1\right]\Big|_{x=2}$

$= 0 + 1$

$= RHS$ verified $y(2)=1$

1.2-5

Solve $\frac{dy}{dx} = \frac{1}{\sqrt{x+2}}$, $y(2) = -1$.

$y'(x) = \frac{1}{\sqrt{x+2}}$ Given DE

$\int y'(x)\,dx = \int \frac{dx}{\sqrt{x+2}}$ Apply method of quadrature

$y(x) = \int u^{-1/2}\,du$ Let $u = x+2$

$y(x) = 2u^{1/2} + c$ power rule

$y(x) = 2\sqrt{x+2} + c$

$-1 = 2\sqrt{2+2} + C$ use $y(2) = -1$

$C = -5$

$y(x) = 2\sqrt{x+2} - 5$

check: agrees with textbook

See also: J. Lahti Slide 1.2-5