Name KEY

Differential Equations and Linear Algebra 2250

Midterm Exam 2

Version 1a, Thu 29 March 2012

Scores
4.
5.

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

4. **(Determinants)** Do all parts.

(a) [20%] State the Four Rules for computing the value of any determinant.

(b) [20%] Assume given 3×3 matrices A, B. Suppose $AB = E_3E_2E_1A$ and E_1, E_2, E_3 are elementary matrices representing respectively a swap, a combination, and a multiply by $-1/3$. Assume $\det(A) = 4$. Find $\det(2B)$.

(c) [20%] Determine all values of x for which B^{-1} fails to exist, where B equals the transpose of the matrix $\begin{pmatrix} 2 & 0 & 2x & 0 \\ 3x & 0 & 10 & 0 \\ 1 & 2x-1 & 7 & 0 \\ x^4 & x^3 & x^2 & x \end{pmatrix}$.

(d) [40%] Apply the adjugate [adjoint] formula for the inverse to find the value of the entry in row 2, column 3 of A^{-1}, given A below. Other methods are not acceptable.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 1 \\ 1 & 2 & 0 & 2 \end{pmatrix}$$

(a) triangular rule, swap rule, multiply rule, combo rule
(statements omitted here, but expected on exam papers – This is a key)

(b) $|A|\,|B| = |E_3|\,|E_2|\,|E_1|\,|A| = (-\frac{1}{3})(1)(-1)\,|A|$ by the product rule for determinants and elementary matrix determinant identities. Then $|B| = \frac{1}{3}$. Because $|2B| = |2I|\,|B| = 2^3\,|B|$, then $|2B| = 8/3$

(c) B^{-1} fails to exist $\Leftrightarrow |B| = 0 \Leftrightarrow x(-1)(2x-1)\begin{vmatrix} 2 & 2x \\ 3x & 10 \end{vmatrix} = 0$
$\Leftrightarrow x = 0,\ x = 1/2,\ x = \pm\sqrt{\frac{20}{6}}$

(d) $\text{entry} = \dfrac{\text{cofactor}(A,3,2)}{|A|} = \dfrac{(-1)^{3+2}\,\text{minor}(A,3,2)}{|A|} = \dfrac{(-1)(-2)}{1} = 2$

$$|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -2 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1$$

combos

$$\text{minor}(A,3,2) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & -2 \\ 1 & 0 & 2 \end{vmatrix} = (-2)(-1)\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -2$$

Use this page to start your solution. Attach extra pages as needed.

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5. **(Linear Differential Equations)** Do all parts.

(a) [20%] Solve for the general solution of $20y'' + 9y' + y = 0$.

(b) [40%] The characteristic equation is $r(r-3)^2(r^2+2r+10) = 0$. Find the general solution y of the linear homogeneous constant-coefficient differential equation.

(c) [20%] A second order linear homogeneous differential equation with constant coefficients has two particular solutions $e^{3x}\sin 2x$ and $e^{3x}(2\sin 2x + 3\cos 2x)$. What are the roots of the characteristic equation?

(d) [20%] **Circle the functions** which can be a solution of a linear homogeneous differential equation with constant coefficients. For example, you would circle $\cos^2 x$ because $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$ is a linear combination of the two solutions 1 and $\cos(2x)$ of a third order equation whose characteristic equation has roots 0, $2i$, $-2i$.

$e^{\ln|2x|}$ $\quad e^{x^2}$ $\quad e+1$ $\quad \cos(\ln|x|)$ $\quad \tan x$

$\cos(x\ln|3.7125|)$ $\quad x^{10}e^{-x}\sin(\pi x)$ $\quad \sinh x$ $\quad \sin^2 x$ $\quad \sin(x^2)$

(a) $(4r+1)(5r+1)=0$, atoms $= e^{-x/4}, e^{-x/5}$, $y =$ linear combination of the 2 atoms

(b) $r^2+2r+10 = (r+1)^2+9$, roots $= 0, 3, 3, -1\pm 3i$, atoms $= 1, e^{3x}, xe^{3x}, e^{-x}\cos 3x, e^{-x}\sin 3x$, $y =$ linear combination of the atoms

(c) $e^{3x}\cos 2x$ comes from roots $\boxed{3\pm 2i}$. Both solutions are l.c. of the atoms from roots $3\pm 2i$.

(d) $e^{\ln|2x|} = |2x|$ not an atom; e^{x^2} not an atom; $e+1 =$ l.c. of atoms, $= c\cdot 1$ where $c = e+1 = 2.718+1 = 3.718$; $\cos(\ln|x|)$ not an atom; $\tan x$ not an atom; $\cos(bx)$ with $b = \ln|3.7125|$ is an atom; $x^{10}e^{-x}\sin(\pi x)$ is an atom; $\sinh(x) = \frac{e^x}{2} + (-\frac{1}{2})e^{-x}$ is a l.c. of atoms e^x, e^{-x}; $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$ is a l.c. of atoms; $\sin(x^2)$ not an atom.

Use this page to start your solution. Attach extra pages as needed.