Name KEY 1a, ver 2

Scores
4.
5

Differential Equations and Linear Algebra 2250

Midterm Exam 1a
Version 2, 23 Feb 2012

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

4. (Linear Equations)

(a) [50%] Solve the linear model. Show all integrating factor steps.

$$\begin{cases} 3x'(t) &= -48 + \dfrac{12}{2t+5}x(t), \\ x(0) &= 40 \end{cases}$$

(b) [20%] Solve the homogeneous equation $\dfrac{dy}{dx} - (2x-1)y = 0$.

(c) [30%] Solve $13\dfrac{dy}{dx} + 26y = \dfrac{7}{2}$ using the superposition principle $y = y_h + y_p$. Expected are answers for y_h and y_p.

(a) $x' - \frac{4}{2t+5}x = \frac{-48}{3} \Rightarrow \frac{(xw)'}{w} = -16 \Rightarrow xw = c - 16\int w\,dt$

$w = e^{\int p\,dt} = e^{-2\ln|2t+5|+c_1}$; choose $w = (2t+5)^{-2}$. Then

$\int w\,dt = \frac{-1}{2}(2t+5)^{-1}$ and $xw = c + 8(2t+5)^{-1}$,

$x = c(2t+5)^{-2} + 8(2t+5)$. Because $x(0)=40$, Then

$40 = c/25 + 40 \Rightarrow c = 0$ and $\boxed{x = 16t + 40}$

(b) The answer is $y = \dfrac{c}{\text{integrating factor}} = \boxed{\dfrac{c}{e^{-x^2+x}}}$

(c) Choose y_p: $0 + 26y = \frac{7}{2} \Rightarrow \boxed{y_p = \frac{7}{52}}$

Compute $y_h = \dfrac{c}{\text{integr. factor}} = \dfrac{c}{e^{2x}} \Rightarrow \boxed{\begin{array}{l} y_h = ce^{-2x} \\ y = y_h + y_p \end{array}}$

Use this page to start your solution. Attach extra pages as needed.

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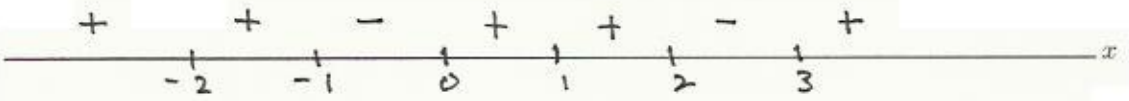
5. (Stability)

(a) [50%] Draw a phase line diagram for the differential equation

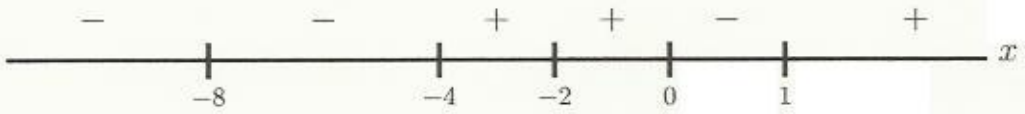
$$\frac{dx}{dt} = \sinh(x)\,(2 - |2x-4|)^3\,(2+x)(x^2-4)(1-x^2)^3$$

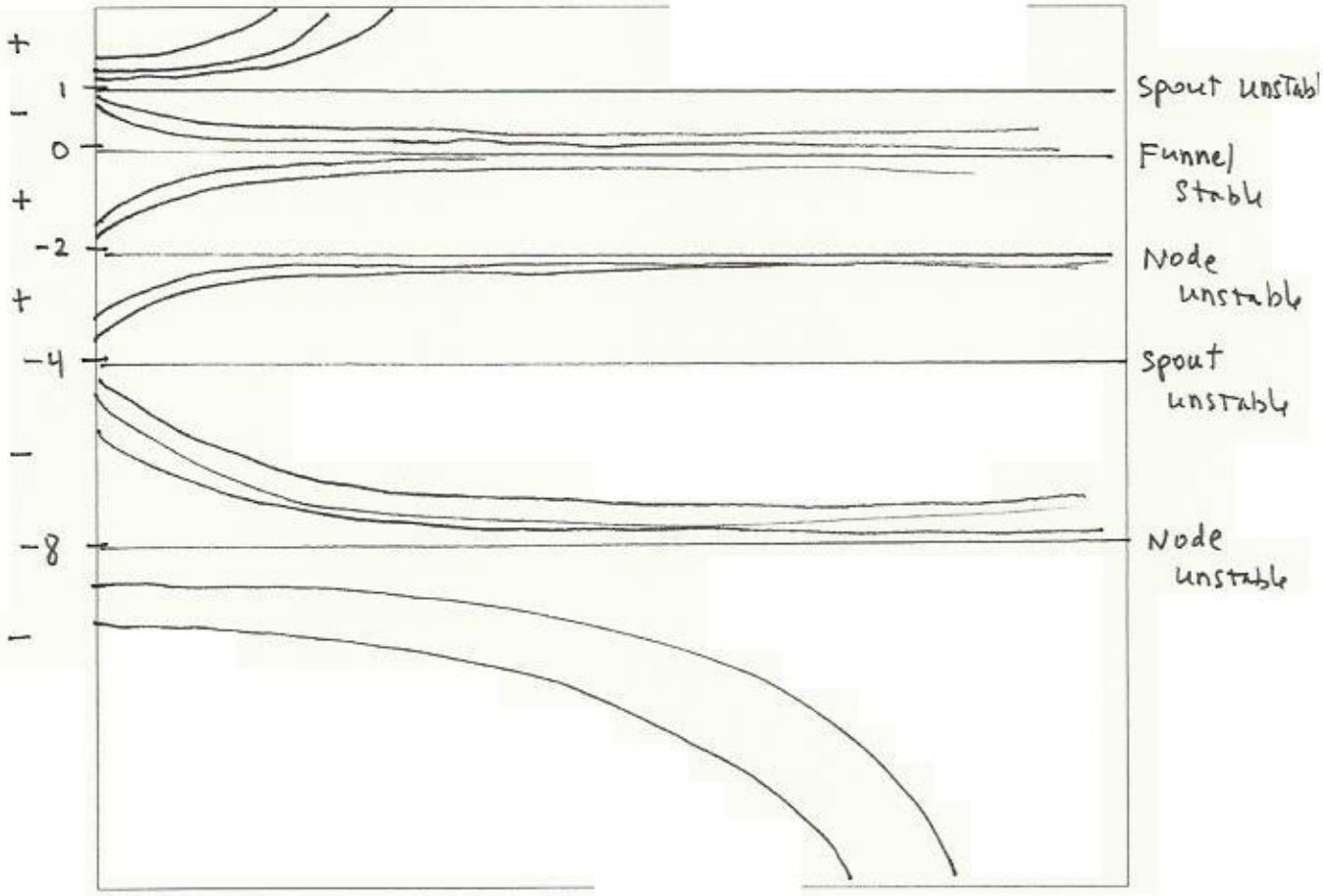
Expected in the phase line diagram are equilibrium points and signs of dx/dt. Definition: $\sinh(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$\sinh(x) = 0$ at $x=0$; $2-|2x-4| = 0 \iff 2x-4 = \pm 2 \iff x = 3$ or $x = 1$; other roots are $-2, -2, 2, 1, -1$. There are 6 roots



(b) [50%] Assume an autonomous equation $x'(t) = f(x(t))$ Draw a phase diagram with at least 12 threaded curves, using the phase line diagram given below. Add these labels as appropriate. **funnel**, **spout**, **node** [neither spout nor funnel], **stable**, **unstable**.





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