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> *restart*

> *# lab 2 problem with a different model. created with maple 12. feel free to email any corrections (be wary of typos!) or suggestions. keep in mind this is meant to help you understand the lab and maple better, but you still have to do your own thinking.*

> *model := diff(u(t), t) + k·u(t) = sin(t);*

$$model := \frac{d}{dt} u(t) + k\, u(t) = \sin(t) \tag{1}$$

> *dsolve({model, u(0) = u0})  #model solved using dsolve and matched against solution by hand*

$$u(t) = e^{-kt}\left(u0 + \frac{1}{k^2 + 1}\right) + \frac{-\cos(t) + \sin(t)\, k}{k^2 + 1} \tag{2}$$

> *# solving model using integrated factor approach*
> *integratingFactor := exp(int(k, t));*

$$integratingFactor := e^{kt} \tag{3}$$

> *u := (u0 + int(sin(x)·exp(k·x), x = 0..t))·exp(-k·t);*

$$u := \left(u0 - \frac{-1 + e^{kt}\cos(t) - k\, e^{kt}\sin(t)}{k^2 + 1}\right) e^{-kt} \tag{4}$$

> *# note homogenous solution for this will have form C·exp(−k·t), and everything else is with the particular solution, and u0 = u(0)*

> *# indentify uh and up using this*

> *uh := (u0 + \frac{1}{k^2 + 1})·exp(-k·t);*

$$uh := e^{-kt}\left(u0 + \frac{1}{k^2 + 1}\right) \tag{5}$$

> *up := - \frac{(cos(t) - k·sin(t))}{k^2 + 1};*

$$up := -\frac{\cos(t) - \sin(t)\, k}{k^2 + 1} \tag{6}$$

> *u := up + uh;*

$$u := -\frac{\cos(t) - \sin(t)\, k}{k^2 + 1} + e^{-kt}\left(u0 + \frac{1}{k^2 + 1}\right) \tag{7}$$

> *# verifying the solution*
> *LHS := diff(u, t) + k·u*

$$LHS := -\frac{-\sin(t) - \cos(t)\, k}{k^2 + 1} - k\, e^{-kt}\left(u0 + \frac{1}{k^2 + 1}\right) + k\left(-\frac{\cos(t) - \sin(t)\, k}{k^2 + 1} + e^{-kt}\left(u0 \right.\right. \tag{8}$$

$$\left.\left. + \frac{1}{k^2 + 1}\right)\right)$$

> *RHS := sin(t)*

$$RHS := \sin(t) \tag{9}$$

> *simplify(LHS − RHS)*

$$0 \tag{10}$$

> *simplify(subs({t = 0}, u))  # checking for u(0) = u0*

$$\textbf{(11)}$$

$$-\frac{\cos(0) - \sin(0)\,k - u0\,k^2 - u0 - 1}{k^2 + 1} \tag{11}$$

> 
> 
> 
> 
> # *since for uss, all terms with exp(−k·t) are dropped, therefore*
> $uss := -\dfrac{(\cos(t) - k\cdot\sin(t))}{k^2 + 1};$

$$uss := -\frac{\cos(t) - \sin(t)\,k}{k^2 + 1} \tag{12}$$

> # *now uss is to be turned into a function*
> # *let's unassign uss first, so we can turn uss into a function rather than a symbol*
> $unassign(\,'u',\,'uss'\,);$
> $u := unapply(uh + up,\,(t,\,k,\,u0)\,);$

$$u := (t,\,k,\,u0) \rightarrow -\frac{\cos(t) - \sin(t)\,k}{k^2 + 1} + e^{-kt}\left(u0 + \frac{1}{k^2 + 1}\right) \tag{13}$$

> $uss := unapply(up,\,(t,\,k,\,u0)\,);$

$$uss := (t,\,k,\,u0) \rightarrow -\frac{\cos(t) - \sin(t)\,k}{k^2 + 1} \tag{14}$$

> # *this was part 1 and 2 (almost)*
> # *now for 3 − suppose the outdoor temperature here is sin(t) (our A(t) if you are following the lab writeup), and the indoor temperature is u(t)*
> # *we need to plot sin(t) against our A(t) − first let's unassign everything just to be careful*
> $unassign(\,'k','t',\,'u0','A')$
> $A := unapply(\sin(t),\,(t)\,);$ # *now A is a function of t*
$$A := t \rightarrow \sin(t) \tag{15}$$

> # *now by defintion of u(t) in terms of uss, we need uss(0)*
> 
> $uss0 := uss(0,\,k,\,u0);$ # *just evaluate the function we made for uss at t = 0*
$$uss0 := -\frac{1}{k^2 + 1} \tag{16}$$

> # *we have u(t) = (u(0)−uss(0))·exp(−k·t) + uss(t), turning this into a function, but we need to have uss as a symbol to be able to use unapply*
> # *turning uss into a symbol again by resetting it to up*
> $unassign(\,'uss')$
> $uss := up$
$$uss := -\frac{\cos(t) - \sin(t)\,k}{k^2 + 1} \tag{17}$$

> $U := unapply(\,(u0 - uss0)\cdot\exp(-k\cdot t) + uss,\,(u0,\,k,\,t)\,);$
$$U := (u0,\,k,\,t) \rightarrow -\frac{\cos(t) - \sin(t)\,k}{k^2 + 1} + e^{-kt}\left(u0 + \frac{1}{k^2 + 1}\right) \tag{18}$$

> *# now we have everything in terms of functions, let's plot!!!*
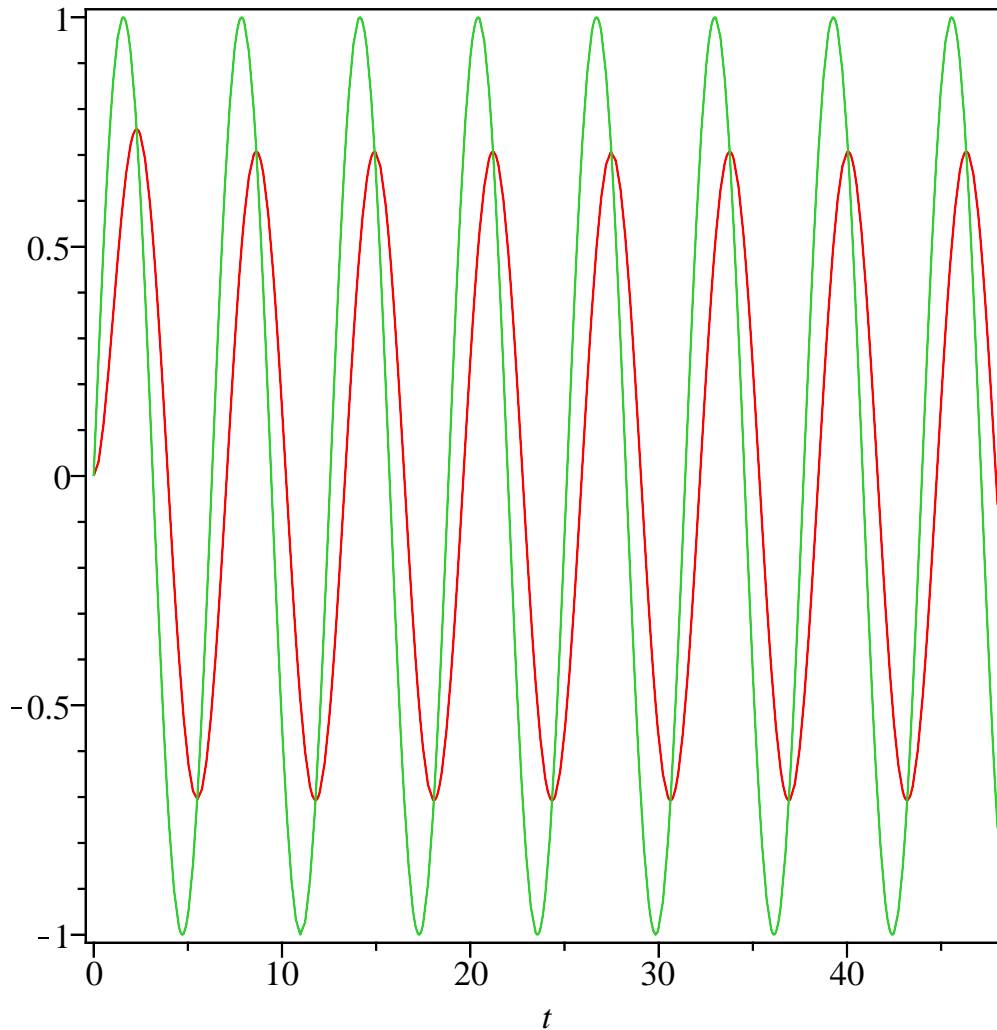> *# let's use k = 1 and u0 = 1*

> $k := 1 ; u0 := 0;$

$$k := 1$$
$$u0 := 0$$

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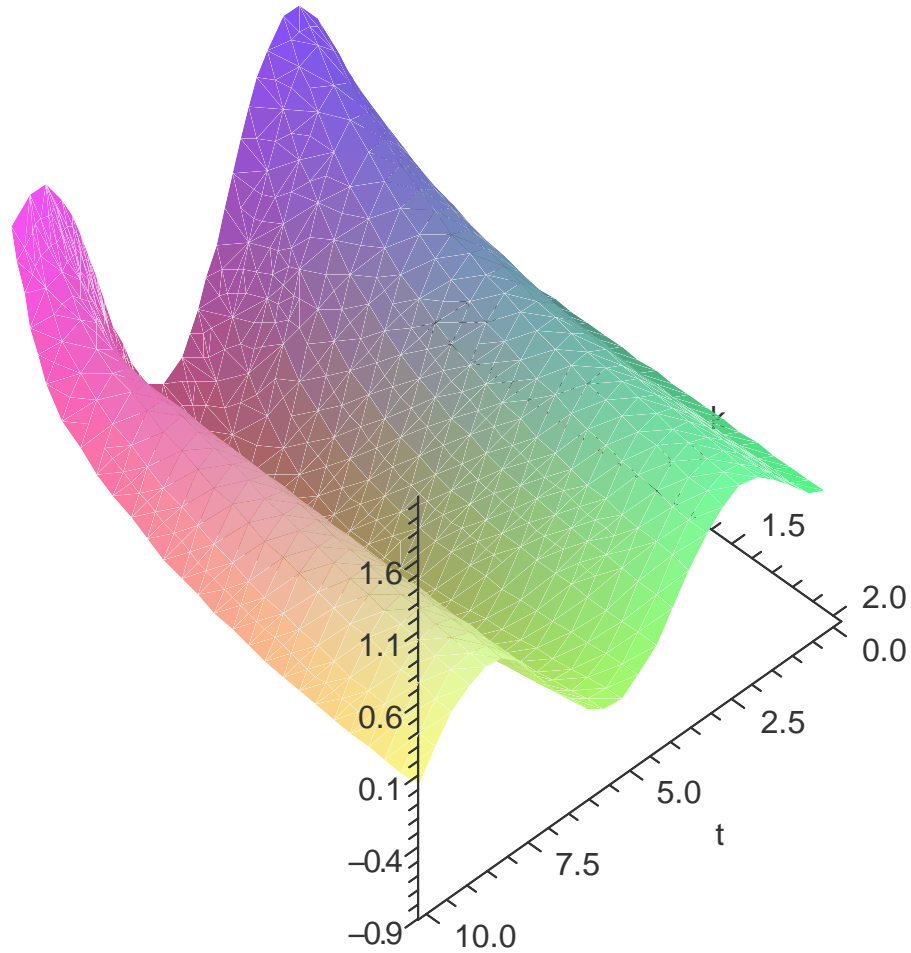
> $plot( \{ U(u0, k, t), A(t) \}, t = 0..48, axes = boxed);$



> *# what we need for this part of lab is the time lag between peaks of A(t) and u(t). either zoom into this graph, or print out the (x,y) pairs for a curve near a max, and then read off the peak time from that.*
> *# now lets see how we can have a 3−d plot allowing us to vary both k and t*
> $with(plots):$
> $unassign('k');$
> $plot3d( \{ U(u0, k, t) \}, t = 0..10, k = 0..2, \ axes = framed, style = patchnogrid);$

> # *you have to draw your own conclusions from looking at this surface generated by u(t) as both*
>   *k and t are varied. and also using implicit plot!*
> # *see maple help on that.*
> # *an example on using implicit plot*
> $u0 = 0$ :
> $unassign('U')$ # *need an expresison for implicit plot*
> $U := unapply((u0 - uss0) \cdot \exp(-k \cdot t) + uss, (k, t))$

$$U := (k, t) \rightarrow -\frac{\cos(t) - \sin(t)\, k}{k^2 + 1} + \frac{e^{-t k}}{k^2 + 1}$$

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> $implicitplot(U(k, t) = 1, t = 0..10, k = 0..1)$ #*plots the implicitly defined curve U(k,t)*
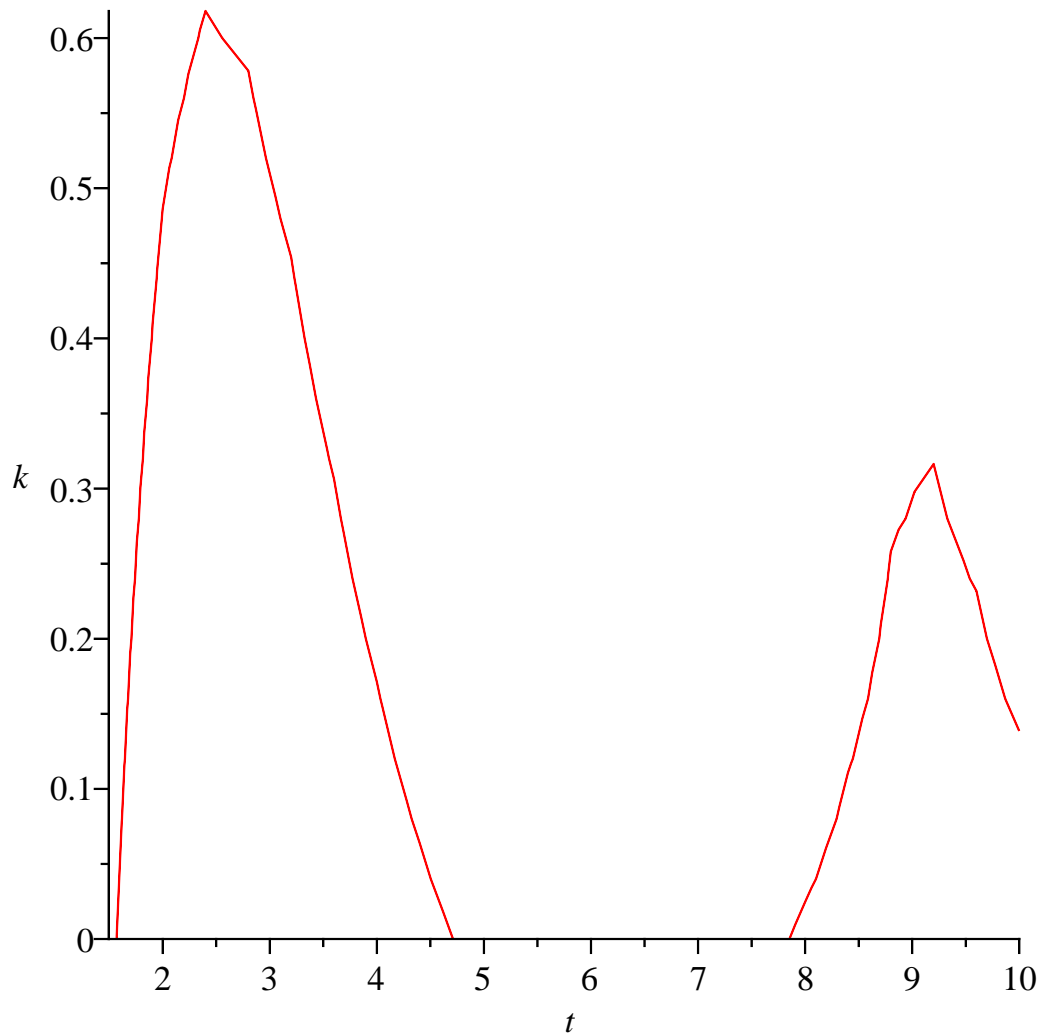
> # implicit plots gives the values of k, t that give U(k,t) = 1, for example note that k = 0.5 and t = 2 looks like a point on this curve

> evalf ( U ( .5, 2 ) ) # almost 1

$$0.9909399929 \tag{21}$$

> # though what you need for your problem on the lab is not U(k,t) = C but U(k,t) > C !!