$\boxed{1.2-5}$

PROBLEM 3 $\boxed{\text{pg 17 \#5}}$ Jennifer L...

1.2°

Find a function $y=f(x)$ which satisfies the given differential equation $\frac{dy}{dx}=\frac{1}{\sqrt{x+2}}$ and the initial condition $y(2)=-1$

$y'(x)=\frac{1}{\sqrt{x+2}}$ same procedure as problems #1 & 2

$\int y'(x)\,dx = \int \frac{dx}{\sqrt{x+2}}$

$y(x)=\int u^{-1/2}\,du$ apply "u" substitution

$y(x)=2u^{1/2}+c$

$y(x)=2\sqrt{x+2}+c$

$-1=2\sqrt{2+2}+c$

$c=-5$

$y(x)=2\sqrt{x+2}-5$

Check: candidate solution agrees with solution given in book.

1.2-5

Solve $\frac{dy}{dx} = \frac{1}{\sqrt{x+2}}$, $y(2) = -1$.

$y'(x) = \frac{1}{\sqrt{x+2}}$	Given DE
$\int y'(x)\,dx = \int \frac{dx}{\sqrt{x+2}}$	Apply method of quadrature
$y(x) = \int u^{-1/2}\,du$	Let $u = x+2$
$y(x) = 2u^{1/2} + c$	power rule
$y(x) = 2\sqrt{x+2} + c$	
$-1 = 2\sqrt{2+2} + C$	use $y(2) = -1$
$C = -5$	
$y(x) = 2\sqrt{x+2} - 5$	

check: agrees with textbook

See also: J. Lahti slide 1.2-5

1.2-8

PROBLEM 4: Pg. 17 #8 Jennifer Lahti

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Find a function $y = f(x)$ which satisfies the given differential equation $\frac{dy}{dx} = \cos 2x$ and initial condition $y(0)=1$

$y'(x) = \cos 2x$ apply same procedure as in problem #5

$\int y'(x)\,dx = \int \cos 2x\,dx$

$y(x) = \frac{1}{2}\sin u + c$

$y(x) = \frac{1}{2}\sin 2x + c$

$1 = \frac{1}{2}\sin 2(0) + c$

$y(x) = \frac{1}{2}\sin 2x + 1$ candidate solution

Check:

$LHS = y'(x)$

$= (\frac{1}{2}\sin 2x + 1)'$

$= (\frac{1}{2})(2)\cos 2x + 0$

$= RHS$ checks with initial differential equation.

$LHS = y(0)$

$= \frac{1}{2}\sin 2(0) + 1$

$= 0 + 1$

$= RHS$ checks with initial condition $y(0)=1$

1.2 — #8

Solve $\frac{dy}{dx} = \cos 2x$, $y(0)=1$

$y'(x) = \cos 2x$ Repeat problem 5 methods

$y'(x)\,dx = \int \cos 2x \, dx$

$y(x) = \frac{1}{2}\sin u + C$ $u = 2x$

$y(x) = \frac{1}{2}\sin 2x + C$

$1 = \frac{1}{2}\sin(0) + C$ Use $y(0)=1$

$y(x) = \frac{1}{2}\sin 2x + 1$ Candidate solution

Check:

LHS $= y'(x)$

$= \left(\frac{1}{2}\sin 2x + 1\right)'$

$= \left(\frac{1}{2}\right)(2)\cos 2x + 0$

$=$ RHS DE $y' = \cos 2x$ verified

LHS $= y(0)$

$= \frac{1}{2}\sin(0) + 1$

$= 0 + 1$

$=$ RHS IC $y(0)=1$ verified

1.2-10

PROBLEM 5: $\boxed{\text{pg. 17 \#10}}$ Jennifer Lahti

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Find a function $y=f(x)$ which satisfies the given differential equation $\frac{dy}{dx} = xe^{-x}$ and initial condition $y(0)=1$.

$y'(x) = xe^{-x}$

$\int y'(x)\,dx = \int xe^{-x}\,dx$ apply method of quadrature

$y(x) = (-x-1)e^{-x} + C$ integral tables #46

$y(x) = C - xe^{-x} - e^{-x}$

$1 = C - 0(e^0) - e^0$

$C = 2$

$y(x) = 2 - xe^{-x} - e^{-x}$ candidate solution

check:

$LHS = y'(x)$

$= (2 - xe^{-x} - e^{-x})'$

$= 0 - (-x(e^{-x}) + e^{-x}) - (-e^{-x})$ use product rule

$= 0 + xe^{-x} - e^{-x} + e^{-x}$

$= xe^{-x}$

$= RHS$ initial differential equation verified

$LHS = y(0)$

$= 2 - 0(e^0) - e^0$

$= 1$

$= RHS$ initial condition $y(0)=1$ verified.

1.2-10 Apply the method of quadrature to solve

$$\begin{cases} \dfrac{dy}{dx} = x e^{-x}, \\ y(0)=1 . \end{cases}$$

$y = 2 - e^{-x} - x e^{-x}$	The answer, justified below.
$y = \int_0^x y' dx + y(0)$	Fundamental Thm of Calculus $f(b)-f(a) = \int_a^b f' dx$
$= \int_0^x x e^{-x} dx + 1$	use DE + IC
$= 1 - e^{-x} - x e^{-x} + 1$	Integral Tables
$= 2 - e^{-x} - x e^{-x}$	Final answer.

check:

$y(0) = (2 - e^{-x} - x e^{-x}) \Big\|_{x=0}$	
$= 2 - e^{0} - 0$	
$= 1$	IC verified.
$y' = (2 - e^{-x} - x e^{-x})'$	
$= 0 + e^{-x} - e^{-x} + x e^{-x}$	used $(e^{u})' = e^{u} u'$.
$= x e^{-x}$	DE Verified, LHS=RHS.