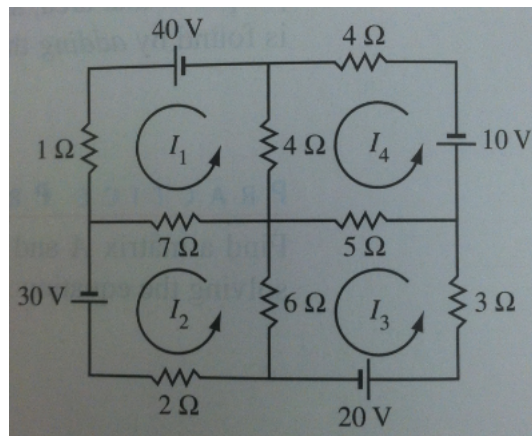


1. (25 points) **Finding Current Flow in a Resistive Network**

Current flow in a simple electrical network can be described by a system of linear equations. By Ohm's law, the "voltage drop" across a resistor is given by $V = RI$ where V represents voltage in volts, R represents resistance in ohms and I represent current flow in amperes. The network depicted in the figure below contains 4 closed loops. The currents flowing in loops 1, 2, 3, and 4 are denoted by I_1 , I_2 , I_3 and I_4 , respectively. If the current direction shown is away from the positive (longer) side of a battery around to the negative (shorter) side, the voltage is positive; otherwise, the voltage is negative.



- (a) (20 points) Using Kirchhoff's Voltage Law, write a matrix equation that determines the loop currents, I_1 , I_2 , I_3 , and I_4 .

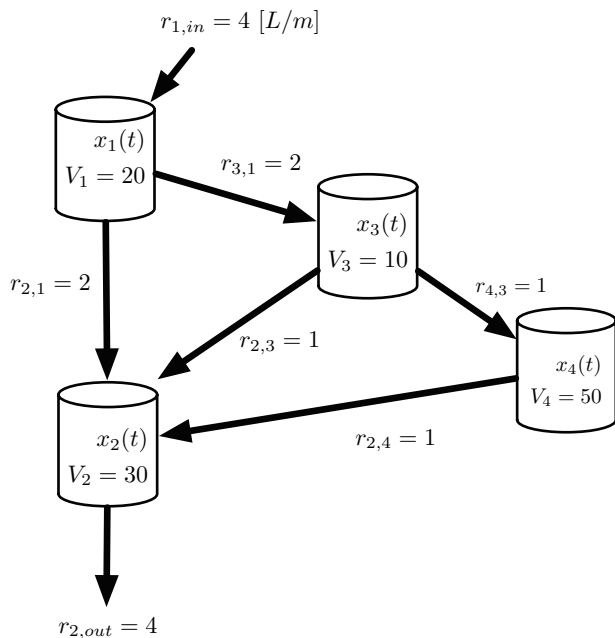
Recall: Kirchhoff's Voltage Law states that the algebraic sum of the RI voltage drops in one direction around a loop is equal to the algebraic sum of the voltage sources in the same direction around the loop.

- (b) (5 points) Use Maple, Matlab or other technology to solve the system for the loop currents, I_1 , I_2 , I_3 , and I_4 .

References: Edwards-Penney Section 3.1-3.4. *Edwards-Penney BVP Section 3.7, electric circuit supplement*. Course documents: *Electric Circuits Slides* and *Linear Applications*. Lay, David C. (2003). *Linear Algebra and its Applications*. Section 1.10: Linear Models in Business, Science and Engineering.

2. (35 points) **Brine Tank Cascade Problem**

This question exhibits a relationship between a system of linear equations and differential equations. Consider the cascade of four tanks depicted in the following figure with the volumes (in L) as $V_1 = 20$, $V_2 = 30$, $V_3 = 10$ and $V_4 = 50$. The flow rates (in L/min) are indicated in the figure (e.g. $r_{2,1}$ is the flow rate from tank 1 to tank 2). The functions $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ (in grams) represent the amount of salt in the tanks 1, 2, 3, and 4 at time t respectively. Suppose the concentration of the solute flowing into tank 1 is 1 g/L.



- (a) (20 points) Express $x_1'(t)$, $x_2'(t)$, $x_3'(t)$ and $x_4'(t)$ in terms of $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$.
- (b) (15 points) Use the expressions in (a) to find a 4×4 matrix \mathbf{A} and a vector \mathbf{v} such that the following relation is satisfied:

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \mathbf{v}.$$

References: Edwards-Penney Sections 1.5, 3.1-3.4, and 7.3. Course WEB notes for cascades and compartment analysis: *System Examples and Theory*.

Some Maple and Matlab Examples

Here the matrix $A = \begin{pmatrix} 2 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ will be used as an example.

When working with matrices in Maple you should first call the *LinearAlgebra* package. This is done by running the command '*with(LinearAlgebra):*'. This will only need to be done once in your Maple document and run it before your commands involving matrices are run.

Creating a matrix

In Maple you would type '*A:=Matrix([[2,2,3],[4,5,6],[7,8,9]])*'.

The *Matrix* command on the right hand side is what tells Maple to turn the data afterwards into a matrix. The *:=* stores that matrix into the variable *A*. Using variables like this is very useful when you plan on using the matrix you have entered more than once since after doing this you can refer to that matrix by simply typing *A*.

To do the same thing in Matlab you would type '*A=[2 2 3; 4 5 6; 7 8 9]*'.

The rest of these examples will be done with the assumption that the matrix is stored into the variable *A*.

Reduced row echelon form

Maple: *ReducedRowEchelonForm(A)*

Matlab: *rref(A)*

Augment the two matrices *A* and *B*

Maple: $\langle A|B \rangle$

Matlab: $[A \ B]$

Computing the inverse

Maple: A^{-1}

Matlab: A^{-1} or *inv(A)*

Creating an $n \times n$ identity matrix

Maple: *IdentityMatrix(n)*

Matlab: *eye(n)*

Computing the determinant

Maple: *Determinant(A)*

Matlab: *det(A)*

Multiplying two matrices *A* and *B*

Maple: $A \cdot B$

Matlab: $A * B$

Reference: Course Document *Introduction to Maple*.