$\mathrm{Math}2250\mathrm{Lab}5$	Name/Unid:		
Due Date: 2/13/2014	Class ID:	Section:	

## 1. (30 points) Balancing Chemical Equations

Chemical balance means that matching elements on each side of a chemical equation have the same total amount of atoms. The balancing of a chemical reaction always leads to a system of linear equations. The equations are often "sparse" enough that the method of substitution will work well, but Gaussian elimination is guaranteed to solve the system.

(a) (15 points) The complete combustion of propane  $C_3H_8$  with oxygen  $O_2$  results in carbon dioxide  $CO_2$  and water  $H_2O$ , in such a way that the total number of atoms of each element are the same before and after the reaction:

$$a(C_3H_8) + b(O_2) \rightarrow c(CO_2) + d(H_2O).$$

The coefficients a, b, c, d for the numbers of each molecule need to be found from the three equations equating the number of C, H, O atoms on each side. For example, equating the number of oxygen atoms yields the equation

$$2b = 2c + d \Rightarrow 2b - 2c - d = 0.$$

Find the other two equations (i.e. find equation for carbon atoms and equation for hydrogen atoms). Then, use Gaussian elimination to find the solutions to this homogeneous system of three equations in four unknowns. Pick the free parameter in the solution so that a, b, c, d are each positive integers having no common factors. This will give you the usual balancing equation for the complete combustion of methane.

(b) (15 points) If there is not enough oxygen for the amount of propane, combustion will be incomplete and the intermediate product carbon monoxide CO will be created but not completely converted into carbon dioxide  $CO_2$ . This is potentially dangerous in home heating. The more complicated partial combustion equation is

$$a(C_3H_8) + b(O_2) \rightarrow c(CO_2) + d(H_2O) + e(CO).$$

If we set a = 1 this equation becomes

$$(C_3H_8) + b(O_2) \rightarrow c(CO_2) + d(H_2O) + e(CO).$$

Solve for c, d, e in terms of the parameter b. Note that all four unknowns must be non-negative, so that b can only be in a certain interval. Discuss the range of outcomes as b varies in this interval.

**References:** Edwards-Penney Section 3.1-3.4. Course document: *Linear Algebraic Equations*. Bretscher, Otto (2005). Linear Algebra with Applications. Chapter 1: Linear Equations. For more information on Carbon Chemistry, consider reading the BBC introduction *Using carbon fuels*.