

Math 2250 Lab 4

Name/Unid: _____

Due Date: 2/6/2014

Class ID: _____ Section: _____

1. (30 points) Leslie Leroy Irvin bails out of an airplane at the altitude of 16,000 ft, falls freely for 20 seconds. Assume linear air resistance kv ft/s² with drag coefficient $\rho=0.18$ without the parachute and $\rho=1.8$ with the parachute.

- (a) Write down the ODE that describes the velocity v before he opens the parachute and solve the IVP.
- (b) Find the velocity v at time 1 sec, the velocity when he just opens the parachute ($t=20$ sec), and the terminal velocity.

Consider the same IVP as in part (a). Apply the following methods to approximate the solution function on the time interval $0 \leq t \leq 1$. Do these computations by hand (using a calculator or other technology to do each step, but not using programmed code). Use programmed code (numericsWN.m) or Maple as a technology answer check.

- (c) Euler Method: use step size $h = 0.5$, i.e. 2 time steps.
- (d) Improved Euler Method: use step size $h = 0.5$ again.
- (e) Runge-Kutta Method: use step size $h = 1.0$, i.e. just 1 time step.
- (f) Compare these approximations for $v(1)$ with the exact solution at $t = 1$ sec and comment about the relative accuracy of the three techniques, by computing the relative errors

$$\left| \frac{v_{approx} - v_{exact}}{v_{exact}} \right|$$

in each case.

Note: The step sizes are so large that you don't expect any of the techniques to be extremely accurate.

References: Edwards-Penney Sections 2.3, 2.4, 2.5, 2.6. Course documents: Numerical Methods Manuscript Section 4.6, Numerical Methods Slides, and Numerical Methods Sample Maple Code. See also: numericsWN.m Matlab file created by Dr. W. Nesse.

2. (40 points) If we drop a package from a helicopter with a parachute attached, the air resistance caused by the parachute is not really linear. Assume for a certain type of parachute the acceleration equation is modeled by

$$\begin{aligned}v'(t) &= 32 - 0.72v - 0.36v^{1.8} \\v(0) &= 0\end{aligned}$$

where velocity has units of ft/s and the package shape and weight are included in the drag details.

- (a) What is the terminal velocity in this model? Notice that the “slope function” $32 - 0.72v - 0.36v^{1.8}$ is a decreasing function of v for $v > 0$, that its value is 32 when $v=0$, and that its limit as $v \rightarrow \infty$ is $-\infty$. Thus, the slope function has exactly one root and you can find it using the Matlab “solve” command or other software.
- (b) This is a differential equation which does NOT have an elementary solution. Use “numericsWN.m” (you may modify the Matlab code according to your need) or another computer algebra system (i.e. Maple) to estimate the solution values at $t=1$ and $t=2$ seconds with the numerical methods Euler, Improved Euler and Runge-Kutta using step sizes $h=0.25$ and $h=0.025$.
- Write the approximate solution for each method with each h and each time $t=1$ and $t=2$.
 - Plot your results and comment about apparent accuracy of the three methods with these different choices of time step size.
- (c) Since we don’t know the exact solution function in this problem, there is not a quick, direct way to check the accuracy of our numerical approximate solutions. There are error estimates that some of you will learn in later numerical analysis classes (analogous to error estimates you may have learned for Taylor series). For the purpose of this lab, undertake the following crude test: Compare the numerical approximations for velocity at $t = 1$ and $t = 2$, for Runge-Kutta with $h = 0.25$ and $h = 0.025$. If these approximations are close, then it’s likely that they are also close to the exact solution.
- (d) Based on your work above, approximately what fraction of the terminal velocity is attained at $t = 1$ and $t = 2$ seconds using the Runge-Kutta Method?
- (e) What does the asymptote on the plot mean? What happens if you change the initial condition to $v(0)= 5$ and $v(0)= 20$?

References: Edwards-Penney Sections 2.3, 2.4, 2.5, 2.6. Course documents: Numerical Methods Manuscript Section 4.6, Numerical Methods Slides, Numerical Methods Sample Maple Code, and Nonlinear Drag Maple Code. See also: numericsWN.m Matlab file created by Dr. W. Nesse.

3. (30 points) The classic *Lotka-Volterra model of competition* between two species which compete for the same resource helps to answer the following question: Under what conditions will two species coexist? Suppose you have two species (i.e. rabbits and sheep) which compete for the same food supply (i.e. grass) and that the amount of this resource is limited. For simplicity, we will ignore other factors such as predation and seasonal effects. A system that represents the population dynamics of these two species is given as follows:

$$\begin{aligned}\frac{dR}{dt} &= R(5 - R - 2S) \\ \frac{dS}{dt} &= S(3 - R - S)\end{aligned}$$

where $R(t)$ represents the population of rabbits at time t , $S(t)$ represents the population of sheep and $R(t), S(t) \geq 0$.

Find the equilibrium points of this system. That is, solve for $\frac{dR}{dt} = 0$ and $\frac{dS}{dt} = 0$ simultaneously. Provide a technology answer check.

References: Edwards-Penney, Section 9.1. Course Documents: Linear Algebraic Equations Manuscript, Introduction to Linear Equations Slides, and Unique Solution Case Slides. See also: Strogatz, Steven H. *Nonlinear Dynamics and Chaos* (1994), Section 6.4.