

1. (25 points) Suppose you wish to dispose of nuclear waste by placing it in sealed drums and dropping the drums into the ocean. Each filled drum weighs 1280 lb and has a volume of 18 cubic feet. The force equation for a drum falling though water is:

$$m\frac{dv}{dt} = -W + B + F_R$$

where W is the weight of the drum, B is the buoyant force equal to the weight of the water displaced by the drum (the density of water is 62.5 pounds per cubic foot), and F_R is the force of water resistance, known to be 1 lb for every ft/s that the drum is moving. The drums may burst if they hit the ocean floor at a speed higher than 120 ft/s.

Note: A technology check is expected for computations. Any claimed numeric answer, symbolic answer or equation should be verified. If a technology answer check is impossible, then provide on paper whatever details are possible.

- (a) Find an expression for the velocity v(t).
- (b) Find the time t at which the drum velocity absolute magnitude is 120 ft/s.
- (c) If no initial velocity is given to the drums, what is the deepest water into which the drums can be dropped without violating the bursting rate 120 ft/s?

References: Edwards-Penney, Sections 1.4, 1.5, 2.3. Course documents: Newton kinematics with air resistance.

2. (25 points) Paratrooper with Linear Drag

A paratrooper bails out of an airplane at altitude 15,000 ft, then falls freely for 30 seconds, then opens a parachute. Assume linear air resistance kv ft/s² with drag coefficient $\rho=0.4$ before the parachute opens and $\rho=4$ after the parachute opens.

Note: A technology check is expected, as in the previous problem.

- (a) Write down the ODE for velocity v of the paratrooper before the parachute opens.
- (b) Find the velocity at time 5 seconds and the velocity at time 30 seconds, when the paratrooper just opens the parachute.
- (c) Find the height when the paratrooper opens the parachute.
- (d) Explain why $w' = -32 4w, w(0) = v(30) \approx -80$ is the model for the paratrooper's descent after the parachute opens.
- (e) Explain why the total time of descent for the paratrooper is t = 30 + 1598 = 1628s.

Reference: Edwards & Penney Section 2.3. This traditional parachute problem can be modified to be more realistic. Read Meade & Struthers (1999) *Differential Equations in the New Millennium: the Parachute Problem*, Int. J. Engng Ed. 15(6), 417-424. Retrieved on January 14, 2014. See also Meade, D (1998), *ODE Models for the Parachute Problem*, SIAM Review. 40(2). 327-332. Retrieved on January 14, 2014 at