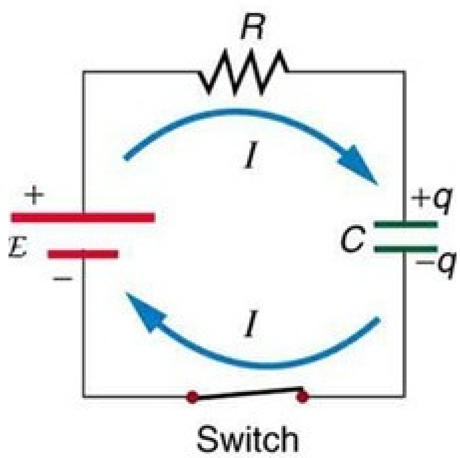
Math 2250 Lab 2 Name/Unid: ______________________________

Due Date: 1/23/2014 Class ID: _______ Section: _______

1. (20 points) **Electric Circuits**

For the RC-circuit shown below, suppose constant input voltage $V(t) = V_0$. In the figure below, V(t) is depicted as E(t), where the E stands for EMF (electromotive force). According to Kirchhoff's current law, $V_R(t) + V_C(t) = V(t)$, where $V_R(t) = I(t)R = RCV_C'(t)$ is the voltage across the resistor and $V_C(t)$ is the voltage across the capacitor.



(a) Show that $V_C(t) = -V_0 e^{\frac{-t}{RC}} + V_0$ given $V_C(0) = 0$.

(b) Assume $V_0 = 200$ volts. Find $V_R(t)$. Provide a technology answer check.

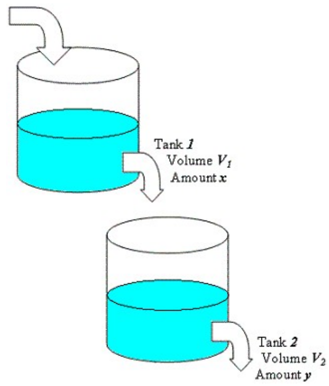
Background: Let $V(t)$ represent input voltage in volts. Then $V(t) \equiv V_0$ represents a constant applied voltage, C represents capacitance in farads, R represents resistance in ohms, and $I(t)$ represents current in amperes. From Kirchhoff's laws, the algebraic sum of the voltage drops is equal to the input voltage $V(t)$. Then, Ohm's Law implies $V_R(t) + V_C(t) = V(t)$ where $V_R(t) = RI(t)$ and Coulomb's Law implies $V_C(t) = \frac{Q(t)}{C}$. The charge $Q(t)$ and the current $I(t)$ satisfy the formula $I(t) = Q'(t) = CV_C'(t)$.

References: Edwards and Penney BVP3.7 (supplement for 2250).

Course WEB notes: Exponential Applications Library 1.2,

Linear Applications, Example 27. Wikipedia reference RC circuit

2. (30 points) **Brine Tank Cascade**
   Consider two brine tanks with water cascading from tank 1 down into tank 2, and then out of tank 2, as in the figure below.



   Suppose that tank 1 initially contains 60 gallons of brine, while tank 2 contains 120 gallons of pure water. When the system starts, fresh water is pumped into tank 1 at a constant rate of 4 gal/min. At the same time, the brine solution in tank 1 drains into tank 2 at a rate of 4 gal/min, and tank 2 drains at an equal rate, causing the volumes in both tanks to remain constant. Assume that the solutions in each tank remain well-mixed, so that although the salt concentrations are changing in time, the concentration of salt leaving each tank equals the average concentration in that tank.

   (a) If the first tank originally contains 40 lbs of salt, then formulate and solve an initial value problem to find the amount $x(t)$ of salt in tank 1 at time t. Provide a technology answer check.

   (b) Suppose that $y(t)$ is the amount of salt in tank 2 at time t. Show first that $\frac{dy}{dt} = \frac{x}{15} - \frac{y}{30}$. Then solve for $y(t)$ using the function $x(t)$ found in part (a). Provide a technology answer check.

References: Edwards and Penney Sections 1.5 and 7.3. Course WEB notes for cascades and compartment analysis: System Examples and Theory

Note: This worksheet has been modified from the original version you received in lab. Problem 3 will no longer be due on Thursday, January 23, but rather on January 30 as part of Worksheet 3.