

Math 2250 Lab 13

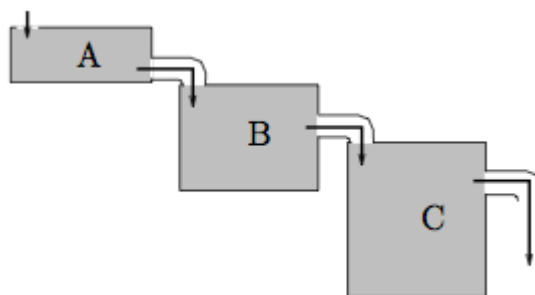
Name/Unid: _____

Due Date: 04/10/2014

Class ID: _____ Section: _____

References: Edwards-Penney: Sections 6.1-6.3, 7.1-7.3, especially Compartmental Analysis in Section 7.3. Course Notes: *Systems of DE examples and theory*. Course Slides: *Algebraic Eigenanalysis* and *What is Eigenanalysis?*

1. (40 points) The figure below depicts three brine tanks in cascade. Suppose brine tanks A, B, and C have volumes, 25, 50, and 75 gallons, respectively. Suppose that fluid enters tank A at rate $r = 15$ gal/min, drains from A to B at rate $r = 15$ gal/min, drains from B to C at rate $r = 15$ gal/min, and drains from tank C at rate $r = 15$ gal/min. Hence, the volumes of each tank remain constant. Assume uniform stirring of each tank so salt concentration is uniform throughout each tank.



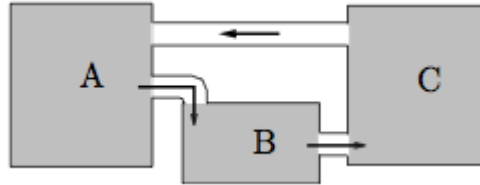
Let $x_A(t)$, $x_B(t)$, $x_C(t)$ denote the amount of salt at time t in tank A, tank B, and tank C, respectively. Suppose water containing no salt is added to tank A. Thus, all salt is eventually lost from the drains.

Recall that the cascade is modeled by the chemical balance law:

$$\text{rate of change} = \text{input rate} - \text{output rate}$$

- (a) Using compartmental analysis, set up the system of differential equations which models the change in the amount of salt in each tank over time.
- (b) Find the general solution for this linear system of equations using eigenanalysis. That is, find the amount of salt, $x_A(t)$, $x_B(t)$, $x_C(t)$, at time t in the three brine tanks.
- (c) Provide a technology answer check for part (b).

2. (60 points) The figure below depicts a recycled brine tank cascade. Suppose brine tanks A, B, and C have volumes 50, 25 and 50 gallons, respectively. Suppose that fluid drains from tank A to B at rate $r = 15$ gal/min, drains from tank B to C at rate $r = 15$ gal/min, then drains from tank C to A at rate $r = 15$ gal/min.



Note: The tank volumes remain constant because of constant recycling of the fluid within the cascade. Uniform stirring of each tank implies uniform salt concentration throughout each tank, and due to recycling, no salt is lost from the system.

Let $x_A(t)$, $x_B(t)$, $x_C(t)$ denote the amount of salt at time t in tank A, tank B, and tank C, respectively.

- Using compartmental analysis, set up the system of differential equations which models the change in the amount of salt in each tank over time.
- Find the general solution for this system of equations using eigenanalysis. That is, find the amount of salt, $x_A(t)$, $x_B(t)$, $x_C(t)$, at time t in the three brine tanks.
- Find the ratio that describes how the amount of salt is uniformly distributed in the tanks as $t \rightarrow \infty$.

