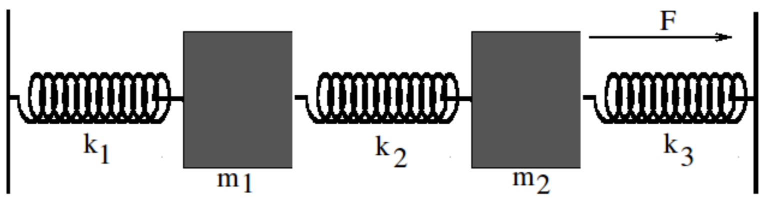
Math 2250 Lab 11 Name/Unid: ______________________________

Due Date: 3/27/2014 Class ID: _______ Section: _______

References: Edwards-Penney Sections 5.4, 10.1-10.3, especially Example 3 in Section 10.2. Modeling appears in sections 7.1, 7.4. Course slides: *Basic Laplace Theory*, *Laplace Rules*, and *Laplace Table Proofs*.

1. **Using Laplace Transforms to Solve a Linear Second Order System**
   Consider a coupled mass-and-spring system as depicted in the figure below and described by the system

$$\begin{cases} 4x'' + 16x - 4y &= 0, \\ 2y'' - \ \ 4x + 6y &= 12\sin 2t \end{cases}$$



A Coupled Spring-Mass System
Symbols $k_1, k_2.k_3$ are Hooke's constants. Symbols m_1, m_2 are masses.
Symbol F is a force acting on mass m_2 with magnitude $f(t) = 12\sin 2t$.
Assumed are values $m_1 = 4, m_2 = 2,\ k_1 = 12, k_2 = 4, k_3 = 2$.

The **purpose of this project** is to solve the system for $x(t), y(t)$ when at $t = 0$ both masses are at rest and in the equilibrium position. The external force $f(t) = 12\sin 2t$ is applied to the second mass m_2 starting at time $t = 0$.

(a) Define $X(s) = \mathcal{L}\{x(t)\}$ and $Y(s) = \mathcal{L}\{y(t)\}$. Display the **Laplace Method** details which transform the differential equations and initial conditions ($x(0) = y(0) = x'(0) = y'(0) = 0$) into the 2×2 system of linear algebraic equations

$$(s^2 + 4)X(s) - Y(s) = 0$$

$$(s^2 + 3)Y(s) - 2X(s) = \frac{12}{s^2 + 4}$$

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Suggestion: Use the corollary *Transforms of Higher Derivatives* in Section 10.2.

(b) Solve the transformed system of part (a) for Laplace transforms $X(s)$ and $Y(s)$, using Cramer's Rule for 2×2 systems of linear algebraic equations.

(c) Find the solution $x(t), y(t)$ by backward table methods and partial fractions. Equivalently, compute $x(t) = \mathcal{L}^{-1}\{X(S)\}$ and $y(t) = \mathcal{L}^{-1}\{Y(S)\}$. A technology answer check is expected.