Name ____

Scores
1.
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Differential Equations and Linear Algebra 2250

Sample Midterm Exam 3

2014

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Chapter 5)** Complete all.

(1a) [50%] The differential equation $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 12x^2 + 6x$ has a particular solution $y_p(x)$ of the form $y = d_1x^2 + d_2x^3 + d_3x^4$. Find $y_p(x)$ by the method of undetermined coefficients (yes, find d_1, d_2, d_3).

(1b) [20%] Given $5x''(t) + 2x'(t) + 4x(t) = 0$, which represents a damped spring-mass system with $m = 5$, $c = 2$, $k = 4$, determine if the equation is over-damped , critically damped or under-damped. **To save time, do not solve for** $x(t)$**!**

(1c) [30%] Given the forced spring-mass system $x''+2x'+17x = 82\sin(5t)$, find the steady-state periodic solution.

Use this page to start your solution. Attach extra pages as needed.

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2. **(Chapter 5)** Complete all.

(2a) [60%] A homogeneous linear differential equation with constant coefficients has characteristic equation of order 6 with roots 0, 0, -1, -1, $2i$, $-2i$, listed according to multiplicity. The corresponding non-homogeneous equation for unknown $y(x)$ has right side $f(x) = 5e^{-x} + 4x^2 + x\cos 2x + \sin 2x$. Determine the undetermined coefficients **shortest** trial solution for y_p.

To save time, do not evaluate the undetermined coefficients and do not find $y_p(x)$! Undocumented detail or guessing earns no credit.

(2b) [40%] Let $f(x) = x^3 e^{1.2x} + x^2 e^{-x}\sin(x)$. Find the characteristic polynomial of a constant-coefficient linear homogeneous differential equation of least order which has $f(x)$ as a solution. To save time, do not expand the polynomial and do not find the differential equation.

Use this page to start your solution. Attach extra pages as needed.

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3. **(Chapter 10)** Complete all parts. It is assumed that you have memorized the basic 4-item Laplace integral table and know the 6 basic rules for Laplace integrals. No other tables or theory are required to solve the problems below. If you don't know a table entry, then leave the expression unevaluated for partial credit.

(3a) [40%] Display the details of Laplace's method to solve the system for $x(t)$. Don't solve for $y(t)$!

$$\begin{aligned} x' &= x + 3y, \\ y' &= -2y, \\ x(0) &= 1, \quad y(0) = 2. \end{aligned}$$

(3b) [30%] Find $f(t)$ by partial fractions, the shifting theorem and the backward table, given

$$\mathcal{L}(f(t)) = \frac{2s^3 + 3s^2 - 6s + 3}{s^3(s-1)^2}.$$

(3c) [30%] Solve for $f(t)$, given

$$\mathcal{L}(e^{2t}f(t)) + 2\frac{d^2}{ds^2}\mathcal{L}(tf(t)) = \frac{s+3}{(s+1)^3}.$$

Use this page to start your solution. Attach extra pages as needed.

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4. (Chapter 10) Complete all parts.

(4a) [60%] Fill in the blank spaces in the Laplace table:

Forward Table

$f(t)$	$\mathcal{L}(f(t))$
t^3	$\dfrac{6}{s^4}$
$e^{-t}\cos(4t)$	
$(t+2)^2$	
$t^2 e^{-2t}$	

Backward Table

$\mathcal{L}(f(t))$	$f(t)$
$\dfrac{3}{s^2+9}$	$\sin 3t$
$\dfrac{s-1}{s^2-2s+5}$	
$\dfrac{2}{(2s-1)^2}$	
$\dfrac{s}{(s-1)^3}$	

(4b) [20%]
Find $\mathcal{L}(f(t))$ from the Second Shifting theorem, given $f(t) = \sin(2t)\mathbf{u}(t-2)$, where $\mathbf{u}$ is the unit step function defined by $\mathbf{u}(t) = 1$ for $t \geq 0$, $\mathbf{u}(t) = 0$ for $t < 0$.

(4c) [20%] Find $f(t)$ from the Second Shifting Theorem, given $\mathcal{L}(f(t)) = \dfrac{s\,e^{-\pi s}}{s^2+2s+17}$.

Use this page to start your solution. Attach extra pages as needed.

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5. **(Chapter 6)** Complete all parts.

(5a) [30%] Find the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 & 1 & 12 \\ -4 & 1 & -3 & 15 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & -2 & 7 \end{pmatrix}$. To save time, **do not** find eigenvectors!

(5b) [30%] Given $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 3 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, which has eigenvalues $1, 2, 2$, find all eigenvectors for eigenvalue 2.

(5c) [20%] Suppose a 3×3 matrix A has eigenpairs

$$\left(2, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}\right), \quad \left(2, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right), \quad \left(0, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right).$$

Display an invertible matrix P and a diagonal matrix D such that $AP = PD$.

(5d) [20%] Assume the vector general solution $\vec{\mathbf{u}}(t)$ of the 2×2 linear differential system $\vec{\mathbf{u}}' = C\vec{\mathbf{u}}$ is given by

$$\vec{\mathbf{u}}(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

Find the matrix C.

Use this page to start your solution. Attach extra pages as needed.