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Differential Equations and Linear Algebra 2250

Midterm Exam 3

April 18, 2014

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Chapter 5)** Complete all.

(1a) [40%] The differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 6x$ has a particular solution $y_p(x)$ of the form $y = d_1x^2 + d_2x^3$. Find $y_p(x)$ by the method of undetermined coefficients (yes, find d_1, d_2).

(1b) [20%] Given $5x''(t) + \beta x'(t) + 4x(t) = 0$, which represents a damped spring-mass system with $m = 5$, $c = \beta > 0$, $k = 4$, determine β such that the system is under-damped.

To save time, do not solve for $x(t)$**!**

(1c) [40%] Given the forced spring-mass system $x'' + 4x' + 17x = 257\sin(4t)$, find the steady-state periodic solution.

Use this page to start your solution. Attach extra pages as needed.

Name. ________________________________

2. (Chapter 5) Complete all.

(2a) [60%] A homogeneous linear differential equation with constant coefficients has characteristic equation of degree 5 with roots 0, $-\sqrt{2}$, $-\sqrt{2}$, $1+3i$, $1-3i$, listed according to multiplicity. The corresponding non-homogeneous equation for unknown $y(x)$ has right side $f(x) = 5e^{-\sqrt{2}x} + 4x^2 + e^x \sin 3x$. Determine the undetermined coefficients **shortest** trial solution for y_p.

To save time, do not evaluate the undetermined coefficients and do not find $y_p(x)$! Undocumented detail or guessing earns no credit.

(2b) [40%] Let $f_1(x) = x^3 e^{-x}$, $f_2(x) = x^2 e^{-x}\sin(x)$. Mark with $\boxed{\textbf{YES}}$ or $\boxed{\textbf{NO}}$ the characteristic equations for which at least of f_1 or f_2 is an Euler solution atom.

☐ $r^2(r+1)^3(r-1)^3 = 0$

☐ $r(r^2-1)^4 = 0$

☐ $(r^2+1)^3(r+1)^4 = 0$

☐ $((r+1)^2+1)^2(r+1)^3 = 0$

Use this page to start your solution. Attach extra pages as needed.

Name. ________________________________

3. **(Chapter 10)** Complete all parts. It is assumed that you have memorized the basic 4-item Laplace integral table and know the 6 basic rules for Laplace integrals. No other tables or theory are required to solve the problems below. If you don't know a table entry, then leave the expression unevaluated for partial credit.

(3a) [40%] Display the details of Laplace's method to solve the system for $x(t)$ and $y(t)$.

$$\begin{aligned} &x' = -2x, \\ &y' = x + y, \\ &x(0) = 1, \quad y(0) = 2. \end{aligned}$$

Graded details: (1) Forward transform; (2) Solve the system for $\mathcal{L}(x), \mathcal{L}(y)$; (3) Backward transform; (4) Answer for both $x(t), y(t)$.

(3b) [20%] Find $f(t)$,

$$\mathcal{L}(f(t)) = \frac{5s^2 - s - 1}{s^2(s-1)}.$$

Graded details: (1) Partial fractions; (2) Backward table

(3c) [40%] Solve for $f(t)$, given

$$\frac{d^2}{ds^2}\mathcal{L}(e^t f(t)) = \frac{2}{(s+1)^3}.$$

Graded details: (1) Laplace theorem(s); (2) Shift theorem; (3) Forward table; (4) Backward table; (5) Answer.

Use this page to start your solution. Attach extra pages as needed.

Name. ________________________________

4. **(Chapter 10)** Complete all parts.

(4a) [60%] Fill in the blank spaces in the Laplace table. Partial credit is given for wrong answers, based on documented steps.

Forward Table

$f(t)$	$\mathcal{L}(f(t))$
t^3	$\dfrac{6}{s^4}$
$e^t \sin(\pi t)$	
$(2t+1)^2$	
$t^2 e^{\pi t}$	

Backward Table

$\mathcal{L}(f(t))$	$f(t)$
$\dfrac{3}{s^2+9}$	$\sin 3t$
$\dfrac{1}{s^2+2s+5}$	
$\dfrac{1}{(3s+1)^2}$	
$\dfrac{s+1}{(s-1)^2}$	

(4b) [40%] Solve by Laplace's method for the solution $x(t)$:

$$x''(t) + 4x(t) = 5e^t, \quad x(0) = x'(0) = 0.$$

Graded details: (1) Transform; (2) Isolate $\mathcal{L}(x(t))$; (3) Partial fractions; (4) Backward table; (5) Answer.

Use this page to start your solution. Attach extra pages as needed.

Name. ________________________________

5. (Chapter 6) Complete all parts.

(5a) [30%] Find the eigenvalues of the matrix $A = \begin{pmatrix} 2 & 3 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ 5 & 11 & 1 & -6 \\ -3 & -7 & 2 & -7 \end{pmatrix}$. To save time, **do not** find eigenvectors!

(5b) [40%] Given matrix $A = \begin{pmatrix} -2 & 0 & 0 \\ -5 & -2 & -5 \\ 5 & 0 & 3 \end{pmatrix}$ and one eigenpair $\left(3, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\right)$, find the other eigenpairs.

Graded details: (1) Eigenvalues; (2) Eigenvector details; (3) Answer.

(5c) [30%] Assume the vector general solution $\vec{\mathbf{u}}(t)$ of the 3×3 linear differential system $\vec{\mathbf{u}}' = A\vec{\mathbf{u}}$ is given by

$$\vec{\mathbf{u}}(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{0t} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

where c_1, c_2, c_3 are arbitrary constants (e^{0t} has exponent zero). Find a matrix multiply formula for the matrix A. Then check the answer

$$A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Use this page to start your solution. Attach extra pages as needed.