Partial Differential Equations in Physics and Engineering

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Superposition

Theorem 1 (Section 3.1)
If U_1, U_2 are two solutions of a linear homogeneous partial differential equation, then any linear combination $u = c_1u_1 + c_2u_2$, where c_1, c_2 are constants, is also a solution.

If in addition u_1 and u_2 satisfy a linear homogeneous boundary condition, then so will $u = c_1u_1 + c_2u_2$.

Example. Consider Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Let $u_1 = x + y$, $u_2 = x^2 - y^2$, which are two harmonic functions. They are solutions of Laplace's equation satisfying the boundary condition $u(0, 0) = 0$. Then $u = 2(x + y) + 3(x^2 - y^2)$ is also a solution of Laplace's equation satisfying the same boundary condition $u(0, 0) = 0$.

Vibration of Strings and the Wave Equation

- Free vibrations
- Forced vibrations
- Rectangular membrane

The Method of Separation of Variables

Solution of the 1-dimensional Wave Equation

A Stretched String with Fixed Edges

D’Alembert’s Method

The 1-Dimensional Heat Equation

Steady-State Heat Problem

Heat Conduction in a Bar: Fourier's Problem

Two-Dimensional Wave Equation: Membrane

Two-Dimensional Heat Equation: Rectangular Plate

Laplace's Equation: Dirichlet Problem

Poisson’s Equation and Eigenfunction Expansions