Math 3150 HW3

2.3.2 (a) $f(x) = x$ if $-p < x < p$, $2p$-periodic, odd function

discont. at $(2k+1)p$, $k \in \mathbb{Z}$.

(b) From problem 2.2.13

$$g(x) = x \quad \text{if } -\pi < x < \pi, \quad 2\pi\text{-periodic}$$

has Fourier series:

$$g(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$$

Now

$$\boxed{f(x) = \frac{p}{\pi} g\left(\frac{\pi}{p}x\right) = \frac{2p}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{p}}$$

At the discontinuity points the Fourier series converges to 0.

2.3.7 (a)

$$f(x) = \begin{cases} -\frac{2}{p}(x - p/2) & \text{if } 0 < x < p \\ -\frac{2}{p}(x + p/2) & \text{if } -p < x < 0 \end{cases}$$

$2p$ periodic odd function discontinuous at $x = kp$, $k \in \mathbb{Z}$.

(b) The sine series coeff of $f(x)$ are:

$$b_n = \frac{2}{p} \int_0^p \left(-\frac{2}{p}\right)(x - p/2) \sin \frac{n\pi}{p} x \, dx$$

$$\stackrel{IBP}{=} \frac{4}{pn\pi} \cos \frac{n\pi}{p} x \, (x - p/2) \Big|_0^p - \frac{4}{pn\pi} \int_0^p \cos \frac{n\pi}{p} x \, dx$$

$$= \frac{4}{pn\pi} \frac{p}{2} \left((-1)^n + 1\right) - \underbrace{\cancel{\frac{4}{pn\pi} \frac{p}{n\pi} \sin \frac{n\pi}{p} x \Big|_{x=0}^{p}}}_{0}$$

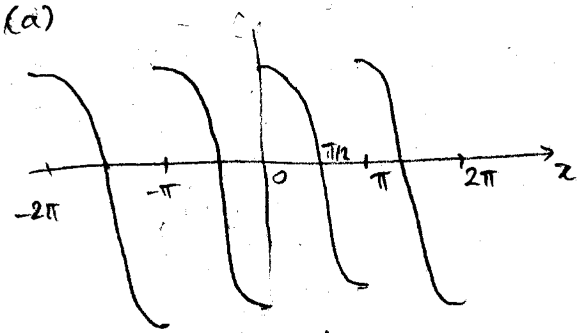
$$= \frac{2}{n\pi} \left((-1)^n + 1\right) = \begin{cases} \frac{4}{n\pi} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

$$\Rightarrow \boxed{f(x) = \sum_{k=1}^{\infty} \frac{4}{2k\pi} \sin \frac{2k\pi}{p} x}$$

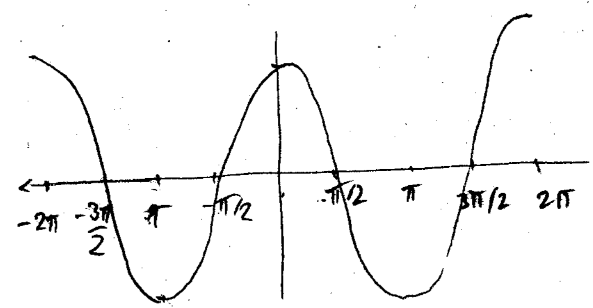
at the discontinuity points the Sine series converges to 0.

2.4.6 $\quad f(x) = \cos x \qquad 0 < x < \pi$.

(a)



odd extension



even extension

Sine series:

$$b_n = \frac{2}{\pi}\int_0^{\pi} \cos x \sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin(n+1)x \, dx + \frac{1}{\pi}\int_0^{\pi} \sin(n-1)x \, dx$$

$$= \frac{-1}{\pi(n+1)} \cos(n+1)x \Big|_0^{\pi} - \frac{1}{\pi(n-1)} \cos(n-1)x \Big|_0^{\pi}$$

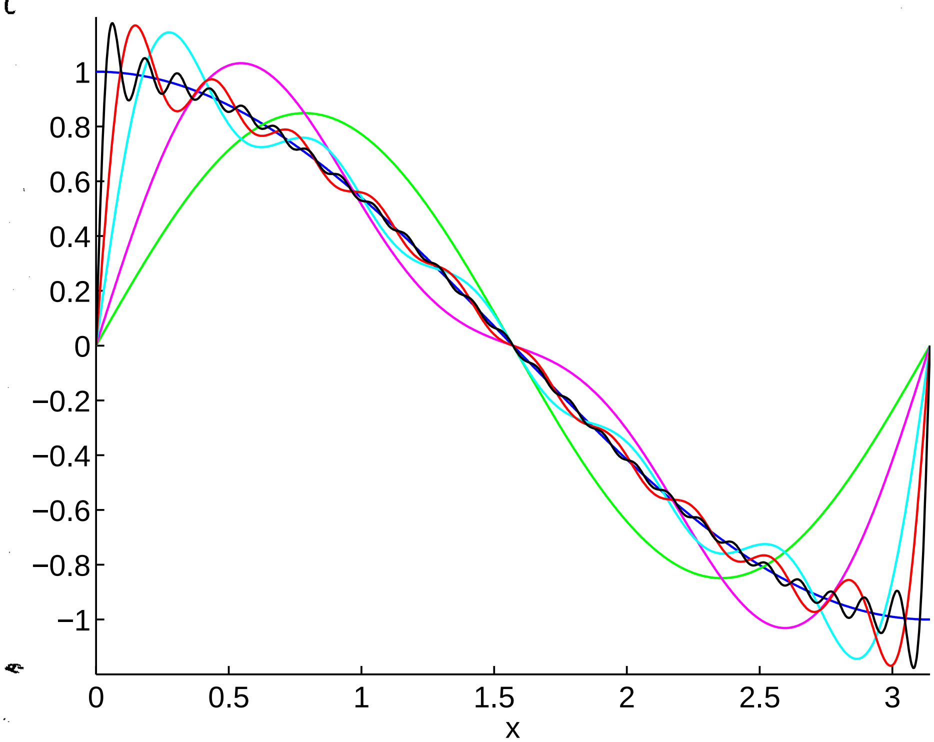
$$= \frac{-1}{\pi(n+1)}\left((-1)^{n+1} - 1\right) - \frac{1}{\pi(n-1)}\left((-1)^{n-1} - 1\right)$$

$$= \begin{cases} \frac{2}{\pi}\left(\frac{1}{n+1} + \frac{1}{n-1}\right) & n \text{ even} \\ 0 & n \text{ odd} \end{cases} = \begin{cases} \frac{4}{\pi}\frac{n}{n^2-1} & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Cosine series:

$f(x) = \cos x$ (function itself)

(plot above).



3.3.2

(a) $f(x) = \sin \pi x \cos \pi x$, $g(x) = 0$, $c = \frac{1}{\pi}$, $L = 1$

Here $g(x) = 0 \Rightarrow b_n^* = 0$ and the solution is:

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin n\pi x \cos nt$$

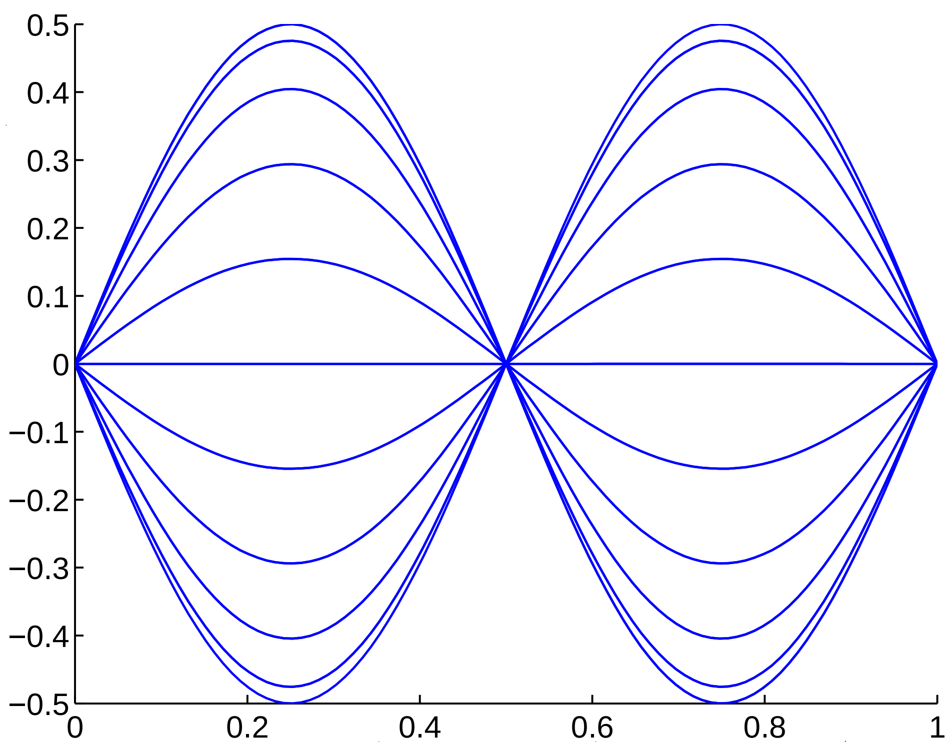
where $b_n = 2\int_0^1 \sin n\pi x \sin \pi x \cos \pi x \, dx$ ↙ double angle formula

$$= \int_0^1 \sin n\pi x \sin 2\pi x \, dx$$

$$= \begin{cases} 1/2 & \text{if } n = 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \boxed{u(x,t) = \frac{1}{2} \sin 2\pi x \cos 2t}$$

(b)



3.3.3

$f(x) = \sin \pi x + 3 \sin 2\pi x - \sin 5\pi x$

$g(x) = 0 \quad \Rightarrow b_n^* = 0$

$c = 1$

$L = 1$

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin n\pi x \cos nt.$$

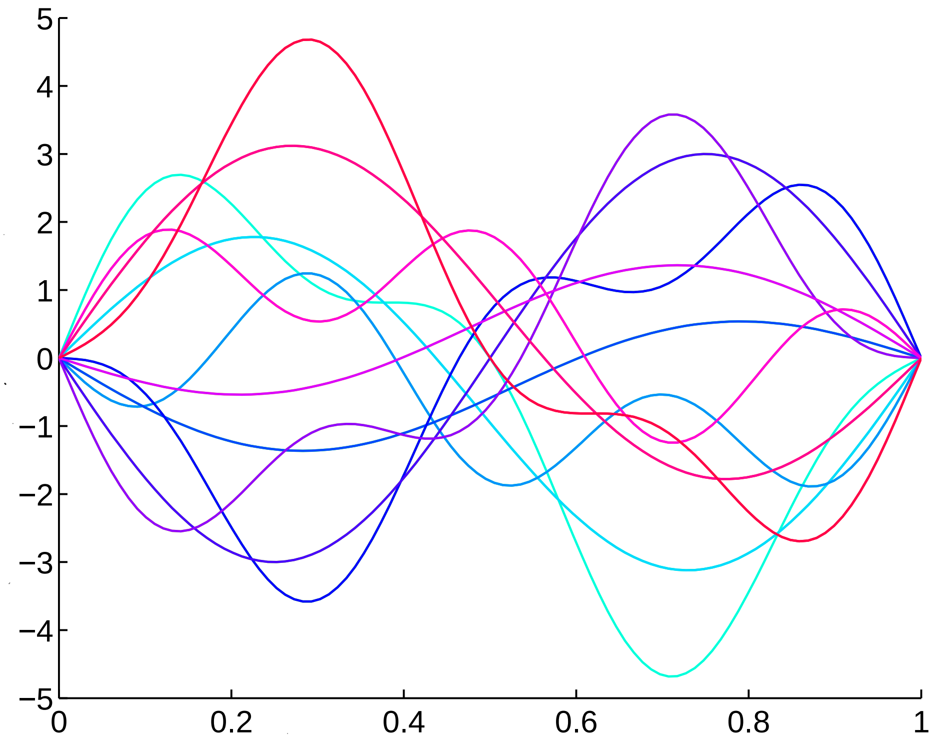
Where $b_n = 2\int_0^1 \sin n\pi x \, f(x)\, dx$.

The Fourier series of f is f itself. $\Rightarrow \begin{cases} b_1 = 1 \\ b_2 = 3 \\ b_5 = -1 \\ b_n = 0 \text{ otherwise} \end{cases}$

and:

$$\boxed{u(x,t) = \sin \pi x \cos t + 3 \sin 2\pi x \cos 2t - \sin 5\pi x \cos 5t}$$

(b)



$\boxed{3.3.4}$ $f(x) = \sin \pi x + \frac{1}{2}\sin 3\pi x + 3\sin 7\pi x$

$g(x) = \sin 2\pi x$

$c = 1$

$L = 1$

The solution to 1DWEQ is:

$$u(x,t) = \sum_{n=1}^{\infty} \sin n\pi x \left(b_n \cos(n\pi c t) + b_n^* \sin(n\pi c t) \right)$$

where $b_n = 2\int_0^1 f(x) \sin(n\pi x)\, dx$

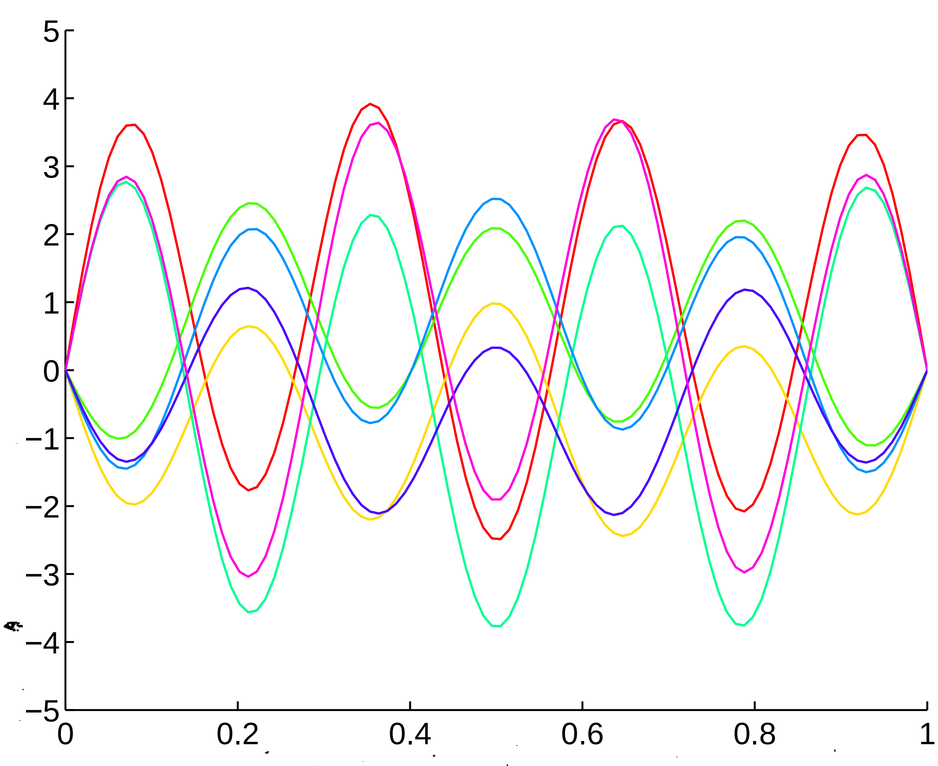
Here Sine Series of f is f itself $\Rightarrow \begin{cases} b_1 = 1 \\ b_3 = 1/2 \\ b_7 = 3 \\ b_n = 0 \text{ otherwise.} \end{cases}$

and $b_n^* = \frac{2}{n\pi}\int_0^1 g(x) \sin(n\pi x)\, dx$

Again sine series of g is g itself $\Rightarrow \begin{cases} b_2^* = \frac{1}{2\pi} \\ b_n^* = 0 \text{ otherwise} \end{cases}$

Thus:

$$\boxed{u(x,t) = \sin \pi x \cos \pi t + \frac{1}{2}\sin 3\pi x \cos 3\pi t + 3 \sin 7\pi x \cos 7\pi t + \frac{1}{2\pi}\sin 2\pi x \cos 2\pi t.}$$



```
% MATH 3150 Fall 2008
% Problem 2.4.6

thickLines(3); % remove if you don't have it in
    your system
figure(1); clf;
x = linspace(0,pi,1000);

% plot true function for refernce
hold on;
plot(x,cos(x));
axis([0,pi,-1.2,1.2]);

% loop over number of terms
Ns = [1,2,5,10,25];
cols={'g','m','c','r','k'};
for iN = 1:length(Ns),
 N=Ns(iN);

 % compute partial Fourier series
 s=zeros(size(x));
 for k=1:N,
  bn = 4*(2*k)/pi/((2*k)^2-1);
  s=s+bn*sin(2*k*x);
 end;

 % comparative plot
 plot(x,s,cols{iN});
end;%N
hold off;
xlabel('x');
filename='p2_4_6.eps';
print('-depsc2',filename);
system(['epstopdf ' filename]);
```

```
% Math 3150 Fall 2008
%
% Problem 3.3.2

thickLines(3); % remove if it is not in your
    system

% space variable
x = linspace(0,1);

% time variable will take values in this array
ts=linspace(0,pi,21);
figure(1); clf;
hold on;
for it=1:length(ts), % for every time
 t = ts(it);
 % we don't need to loop over the number of terms
 % the sum has only ONE term
 s = sin(2*pi*x)*cos(2*t)/2;
 plot(x,s); % plot the partial sum
 % adjust axis so that all plots are on the same
     scale
 axis([0 1 -0.5 0.5]);
end;
hold off;
filename='p3_3_2.eps';
print('-depsc2',filename);
system(['epstopdf ',filename]);
```

```
% Math 3150 Fall 2008
%
% Problem 3.3.3

thickLines(3); % remove if is not in your system

% space variable
x = linspace(0,1);


% time variable will take values in this array
nts = 21; % number of times at which to plot string
ts=linspace(0,2*pi,nts);
cols = hsv(nts); % colors with which to plot string
figure(1); clf;
hold on;
for it=1:length(ts), % for every time
 t = ts(it);
 % we don't need to loop over the number of terms
 % the sum has only a few terms
 s = sin(pi*x)*cos(t) + 3*sin(2*pi*x)*cos(2*t) - sin(5*pi*x)*cos(5*t);
 h=plot(x,s); % plot the partial sum
 set(h,'color',cols(it,:));
 % adjust axis so that all plots are on the same scale
 axis([0 1 -5 5]);
 %pause;
end;
hold off;
filename='p3_3_3.eps';
print('-depsc2',filename);
system(['epstopdf ',filename]);
```

```
% Math 3150 Fall 2008
%
% Problem 3.3.4

thickLines(3); % remove if is not in your system

% space variable
x = linspace(0,1);


% time variable will take values in this array
nts = 7; % number of times at which to plot string
ts=linspace(0,2*pi,nts);
cols = hsv(nts); % colors with which to plot string
figure(1); clf;
hold on;
for it=1:length(ts), % for every time
 t = ts(it);
 % we don't need to loop over the number of terms
 % the sum has only a few terms
 s = sin(pi*x)*cos(pi*t) + (1/2)*sin(3*pi*x)*cos(3*pi*t) + 3*sin(7*pi*x)*cos(7*pi*t) + (1/2/pi)*sin(2*pi*x)*cos(2*pi*t);
 h=plot(x,s); % plot the partial sum
 set(h,'color',cols(it,:));
 % adjust axis so that all plots are on the same scale
 axis([0 1 -5 5]);
 %pause;
end;
hold off;
filename='p3_3_4.eps';
print('-depsc2',filename);
system(['epstopdf ',filename]);
```