The No Solution Case

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No Solution Case

A **signal equation** is a nonzero equation having no variables. It is typically encountered in frame sequences as the equation $\boxed{\mathbf{0 = 1}}$.
When a signal equation occurs in a frame sequence, then we report **no solution**, because a signal equation is a false equation, implying that the system of equations cannot have a solution.

An Example

$$\boxed{\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 4, \\ & - & y & - & 3z & = & -1, \\ & & & & 0 & = & 1. \end{array}}$$

Signal Equation $\mathbf{0 = 1}$.

An Illustration of the No Solution Case

$$\begin{array}{rcrcrcr} & & y & + & 3z & = & 2, \\ x & + & y & & & = & 3, \\ x & + & 2y & + & 3z & = & 4. \end{array}$$

Frame 1. Original system.

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 4, \\ x & + & y & & & = & 3, \\ & & y & + & 3z & = & 2. \end{array}$$

Frame 2.

`swap(1,3)`

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 4, \\ & - & y & - & 3z & = & -1, \\ & & y & + & 3z & = & 2. \end{array}$$

Frame 3.

`combo(1,2,-1)`

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 4, \\ & - & y & - & 3z & = & -1, \\ & & & & 0 & = & 1. \end{array}$$

Frame 4.

Signal Equation $0 = 1$.

`combo(2,3,1)`

The signal equation $\boxed{0 = 1}$ is a false equation, therefore the last frame has no solution. Because the toolkit neither creates nor destroys solutions, then the first frame, which is the original system, has **no solution**.

Perplexing Frames

Values cannot be assigned to any variables in the case of no solution. This can be perplexing, especially in a final frame like

$$\boxed{\begin{aligned} \boldsymbol{x} &= \mathbf{4}, \\ \boldsymbol{z} &= \mathbf{-1}, \\ \mathbf{0} &= \mathbf{1}. \end{aligned}}$$

While it is true that $\boldsymbol{x}$ and $\boldsymbol{z}$ were assigned values, the final signal equation $\mathbf{0} = \mathbf{1}$ is false, meaning any answer is impossible.

There is no possibility to write equations for all variables. There is **no solution**. It is a **tragic error** to claim $\boldsymbol{x} = \mathbf{4}$, $\boldsymbol{z} = \mathbf{-1}$ is a solution.