

**Math 2250 Extra Credit Problems**  
**Chapter 10**  
**S2013**

**Due date:** Submit these problems in week 13. Records are locked on that date and only corrected, never appended. The scores on Ch10 extra credit can replace any missing score any chapter.

**Submitted work.** Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the problems Extra Credit. Label each problem with its corresponding problem number, e.g., Xc10.3-20.

**Problem Xc10.3-20. (Inverse transform)**

Solve for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{1}{s^4 - 8s^2 + 16}$ . Use partial fractions in the details.

**Problem Xc10.3-24. (Inverse transform)**

Solve for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{s}{s^4 + 4a^4}$ , showing the details that give the answer  $f(t) = \frac{1}{2a^2} \sinh at \sin at$

**Problem Xc10.4-12. (Inverse transform, convolution)**

Solve for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{1}{s(s^2 + 4s + 5)}$ . Instead of the convolution theorem, use partial fractions for the details. If you can see how, then check the answer with the convolution theorem.

**Problem Xc10.4-26. (Inverse transform techniques)**

Use the  $s$ -differentiation theorem in the details of solving for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \arctan \frac{3}{s+2}$ . You will need to apply the theorem  $\lim_{s \rightarrow \infty} \mathcal{L}(f(t)) = 0$ .

**Problem Xc10.4-40. (Series methods for transforms)**

Expand in a series, using Taylor series formulas, the function  $f(t) = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ . Then find  $\mathcal{L}(f(t))$  as a series by Laplace transform of each series term, separately. Finally, re-constitute the series in variable  $s$  into elementary functions, namely  $e^{-1/s}$  divided by  $\sqrt{s}$ .

**Problem Xc10.5-6. (Second shifting theorem, Heaviside step)**

Find the function  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{se^{-s}}{s^2 + \pi^2}$ .

**Problem Xc10.5-14. (Transforms of piecewise functions)**

Let  $f(t) = \begin{cases} \cos \pi t & 0 \leq t \leq 2, \\ 0 & t > 2. \end{cases}$  Find  $\mathcal{L}(f(t))$ . Details should expand  $f(t)$  as a linear combination of Heaviside step functions.

**Problem Xc10.5-26. (Sawtooth wave)**

Let  $f(t+a) = f(t)$  and  $f(t) = t$  on  $0 \leq t \leq a$ . Then  $f$  is  $a$ -periodic and has a Laplace transform obtained from the periodic function formula. Show the details in the derivation to obtain the answer  $\mathcal{L}(f(t)) = \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$ .

**Problem Xc10.5-28. (Modified sawtooth wave)**

Let  $f(t+2a) = f(t)$  and  $f(t) = t$  on  $0 \leq t \leq a$ ,  $f(t) = 0$  on  $a < t \leq 2a$ . Then  $f$  is  $2a$ -periodic and has a Laplace transform obtained from the periodic function formula. Derive a formula for  $\mathcal{L}(f(t))$ . The answer to this problem can be found in Edwards-Penney, section 10.5.

**Problem Xc-EPbvp-7.6-8. (Impulsive DE)**

Solve by Laplace methods  $x'' + 2x' + x = \delta(t) - 2\delta(t - 1)$ ,  $x(0) = 1$ ,  $x'(0) = 1$ . Check the answer in `maple` using `dsolve({de,ic},x(t),method=laplace)`.

**Problem Xc-EPbvp-7.6-18. (Switching circuit)**

A passive LC-circuit has battery 6 volts and model equation  $i'' + 100i = 6\delta(t) - 6\delta(t - 1)$ ,  $i(0) = 1$ ,  $i'(0) = 1$ . The switch is closed at time  $t = 0$  and opened again at  $t = 1$ . Solve the equation by Laplace methods and report the number of full cycles observed before the steady-state  $i = 0$  is reached (to two decimal places). Check the answer in `maple` using `dsolve({de,ic},i(t),method=laplace)`.

**End of extra credit problems chapter 10.**