Example. Three Possibilities with Symbol k

Determine all values of the symbol k such that the system below has one of the *Three Possibilities*

- (1) No solution,
- (2) Infinitely many solutions,
- (3) A unique solution.

Display all solutions found.

$$x + ky = 2, \ (2-k)x + y = 3.$$

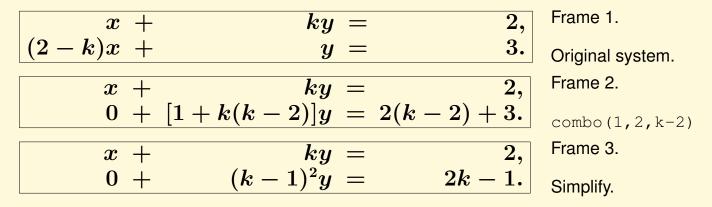
The solution of this problem involves construction of three frame sequences, the last frame of each resulting in one classification among the Three Possibilities:

(1) No solution, (2) Unique solution, (3) Infinitely many solutions.

The plan, for each of the three possibilities, is to obtain a triangular system by application of swap, multiply and combination rules. The three possibilities are detected by (1) A signal equation "0 = 1," (2) One or more free variables, (3) Zero free variables.

Shared Frames

The three expected frame sequences share these initial frames:



At this point, we identify the values of k that split off into the three possibilities.

Analysis of the Frames _

- There will be a signal equation if the second equation of Frame 3 has no variables, but the resulting equation is not "0 = 0." This happens exactly for k = 1. The resulting signal equation is "0 = 1." We conclude that one of the three frame sequences terminates with the *no solution case*. This frame sequence corresponds to k = 1.
- Otherwise, $k \neq 1$. For these values of k, there are zero free variables, which implies a unique solution.
- A by-product of the analysis is that the *infinitely many solutions* case never occurs!

The Conclusion

The three frame sequences reduce to two frame sequences. One sequence gives no solution and the other sequence gives a unique solution.

The Three Answers

(1) There is no solution only for k = 1.

(2) Infinitely many solutions never occur for any value of k.

(3) For $k \neq 1$, there is a unique solution

$$x=2-k(2k-1)/(k-1)^2,\ y=(2k-1)/(k-1)^2.$$