Math 2250 Maple Project 4: Numerical Methods  
S2013

Due date: See the internet due dates. Maple lab 4 has three problems L4.1, L4.2, L4.3.

References: Code in maple appears in 2250mapleL4-S2013.txt at URL http://www.math.utah.edu/~gustafso/. This document: 2250mapleL4-S2013.pdf. Other related and required documents are available at the web site:

- Numerical Solution of First Order DE (typeset, 19 pages, 220k pdf). A resource similar to the textbook, with maple examples and deeper detail. It is for a second reading, in case Edwards-Penney left too many questions unanswered.
- Sample Report for 2.4-3 (pdf 3 pages, 350k). This outline might be useful, if you are wondering which details to include.
- Numerical DE coding hints, TEXT Document (1 page, 2k). A modified portion of this document is appended here, for completeness. The web copy 2250mapleL4-S2013.txt is suited for mouse copying.

Problem Exam Review ER-2. (E & P Exercise 2.4-12, Symbolic Solution)

The exact symbolic solution of $y' = \frac{1}{2}(y - 1)^2, \ y(0) = 2$ is $y = \frac{x - 4}{x - 2}$. Using methods from the textbook, Chapter 1, display the details of the derivation for this symbolic solution, plus a full answer check.

The answer is used in the problems of maple Lab 4, in order to check the calculations.

☐ This problem was submitted earlier, before Exam 1.

☐ This problem was not submitted earlier. The solution will be submitted with maple lab 4.

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Math 2250 Maple Project 4: Numerical Methods
S2013


Problem L4.1. (E & P Exercise 2.4-12)
Consider the initial value problem $y' = \frac{1}{2}(y - 1)^2$, $y(0) = 2$ with symbolic solution $y = \frac{x - 4}{x - 2}$. Apply Euler’s method to produce two dot tables, as follows. The first has 101 rows, $h = 0.01$. The second has 201 rows, $h = 0.005$. Do not print the dot tables, just print the computer code that made them. Reproduce, by transcribing computer data, the table below, and fill in missing digits. Follow the sample report for Exercise 2.4-3.

http://www.math.utah.edu/~gustafso/2250SampleProblem2.4-3.pdf
Reference Exam Review ER-2 for the symbolic solution, excluding any derivation or answer check. Don’t hand-check the dot tables. Instead, test the answers against the symbolic solution, as suggested in the table below. For the percentage error with $h = 0.005$, use the equation

$$\text{Error(approx, actual)} = 100 \frac{|\text{approx} - \text{actual}|}{|\text{actual}|}.$$

<table>
<thead>
<tr>
<th>x</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-approx, $h = 0.01$</td>
<td>2.000000000</td>
<td>2.10466386</td>
<td>2.2482????0</td>
<td>2.4249????1</td>
<td>2.659682??</td>
<td>2.98641???</td>
</tr>
<tr>
<td>y-approx, $h = 0.005$</td>
<td>2.000000000</td>
<td>2.10707534</td>
<td>2.249?????1</td>
<td>2.4267????6</td>
<td>2.66314???0</td>
<td>2.9931????</td>
</tr>
<tr>
<td>actual $y(x)$</td>
<td>2.000000000</td>
<td>2.111111111</td>
<td>2.250000000</td>
<td>2.42571429</td>
<td>2.666666667</td>
<td>3.000000000</td>
</tr>
<tr>
<td>Error(approx,actual)</td>
<td>0.0000%</td>
<td>0.015?%</td>
<td>0.038?%</td>
<td>0.07??%</td>
<td>0.13??%</td>
<td>0.22??%</td>
</tr>
</tbody>
</table>

Problem L4.2. (E & P Exercise 2.5-12)
Consider the initial value problem $y' = \frac{1}{2}(y - 1)^2$, $y(0) = 2$ with symbolic solution $y = \frac{x - 4}{x - 2}$. Apply Heun’s method to produce two dot tables, as follows. The first has 101 rows, $h = 0.01$. The second has 201 rows, $h = 0.005$. Do not print the dot tables, just print the computer code that made them. Reproduce, by transcribing computer data, the table below, and fill in missing digits. Follow the sample report for Exercise 2.4-3 as in problem L4.1 above. Don’t hand-check the dot tables. Instead, test the tables against the symbolic solution, as suggested in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-approx, $h = 0.01$</td>
<td>2.000000000</td>
<td>2.11110???</td>
<td>2.250000000</td>
<td>2.428571428</td>
<td>2.66666???</td>
<td>2.999950375</td>
</tr>
<tr>
<td>y-approx, $h = 0.005$</td>
<td>2.000000000</td>
<td>2.111110682</td>
<td>2.249998780</td>
<td>2.428568700</td>
<td>2.666660895</td>
<td>2.999987544</td>
</tr>
<tr>
<td>actual $y(x)$</td>
<td>2.000000000</td>
<td>2.111111111</td>
<td>2.250000000</td>
<td>2.42571429</td>
<td>2.666666667</td>
<td>3.000000000</td>
</tr>
<tr>
<td>Error(approx,actual)</td>
<td>0.0000%</td>
<td>0.015?%</td>
<td>0.038?%</td>
<td>0.07??%</td>
<td>0.13??%</td>
<td>0.22??%</td>
</tr>
</tbody>
</table>

Problem L4.3. (E & P Exercise 2.6-12)
The actual solution of $y' = \frac{1}{2}(y - 1)^2$, $y(0) = 2$ is $y = \frac{x - 4}{x - 2}$. Apply RK4 method to produce two dot tables, as follows. The first has 101 rows, $h = 0.01$. The second has 201 rows, $h = 0.005$. Do not print the dot tables, just print the computer code that made them. Reproduce, by transcribing computer data, the table below, and fill in missing digits. Follow the sample report for Exercise 2.4-3 as in problem L4.2 above. Don’t hand-check the dot tables. Instead, test the tables against the symbolic solution, as suggested in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-approx, $h = 0.01$</td>
<td>2.000000000</td>
<td>2.111111111</td>
<td>2.250000000</td>
<td>2.428571428</td>
<td>2.666666667</td>
<td>3.000000000</td>
</tr>
<tr>
<td>y-approx, $h = 0.005$</td>
<td>2.000000000</td>
<td>2.111111111</td>
<td>2.250000000</td>
<td>2.428571429</td>
<td>2.666666667</td>
<td>3.000000000</td>
</tr>
<tr>
<td>actual $y(x)$</td>
<td>2.000000000</td>
<td>2.111111111</td>
<td>2.250000000</td>
<td>2.428571429</td>
<td>2.666666667</td>
<td>3.000000000</td>
</tr>
<tr>
<td>Error(approx,actual)</td>
<td>0.0000%</td>
<td>0.0000%</td>
<td>0.0000%</td>
<td>0.0000%</td>
<td>0.0000%</td>
<td>0.0000%</td>
</tr>
</tbody>
</table>

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Warning: These snips of code made for $y' = 1 - x - y$, $y(0) = 3$.
Code computes approx values for $y(0.1)$ to $y(1)$.
'Dots' is the list of dots for connect-the-dots graphics.

---

**Euler. Group 1, initialize.**

```cpp
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:n:=200:
```

**Group 2, repeat n times. Euler's method**

```cpp
for i from 1 to n do
  Y:=y0+h*f(x0,y0):
x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
end;
```

**Group 3, display relevant dots and plot.**

```cpp
Exact:=x->2-x+exp(-x):
P:=unapply(evalf(100*abs(exact-approx)/abs(exact)),(exact,approx)):
m:=40:X:=[seq(1+m*j,j=0..n/m)]: # List of relevant indices
print("Dots"),seq(Dots[k],k=X);
print("Exact"),seq(Exact(Dots[k][1]),k=X);
print("Error"),seq(P(Exact(Dots[k][1]),Dots[k][2]),k=X);
plot([Dots]);
```

---

**Heun. Group 1, initialize.**

```cpp
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:n:=200:
```

**Group 2, repeat n times. Heun method.**

```cpp
for i from 1 to n do
  Y1:=y0+h*f(x0,y0):
  Y:=y0+h*(f(x0,y0)+f(x0+h,Y1))/2:
x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
end;
```

**Group 3, display relevant dots and plot.**

```cpp
Dots[1],Dots[2],seq(Dots[1+40*j],j=1..n/40);
plot([Dots]);
```

---

**RK4. Group 1, initialize.**

```cpp
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:n:=100:
```

**Group 2, repeat n times. RK4 method.**

```cpp
for i from 1 to n do
  k1:=h*f(x0,y0):
  k2:=h*f(x0+h/2,y0+k1/2):
  k3:=h*f(x0+h/2,y0+k2/2):
  k4:=h*f(x0+h,y0+k3):
  Y:=y0+(k1+2*k2+2*k3+k4)/6:
x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
end;
```

**Group 3, display some dots and plot.**

```cpp
Dots[1],Dots[2],Dots[101];
plot([Dots]);
```

---

Code snips for exact/error reports

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Making multiple curves on one plot

```cpp
Exact:=-x->2-x+exp(-x); # An exact solution
plot({Exact(x),[Dots]},x=0..1/2); # plot exact and approx solutions
```

---
# How to create a Dots table for the exact solution
# ========================================
Exact := x -> 2-x+exp(-x): n:=10:
ExactDots := seq([Dots[j][1], Exact(Dots[j][1])], j=1..n+1);

# How to define and print percentage relative error:
# ========================================
P := unapply(evalf(100*abs(exact-approx)/abs(exact)), (exact, approx));
ExactVal := Exact(Dots[11][1]): # Compute exact y-value for x=0.5
ApproxVal := Dots[11][2]:       # Get Euler approx y-value for x=0.5
P(ExactVal, ApproxVal);         # print percent relative error

# How to create a Dots table for percentage error
# ========================================
P := unapply(evalf(100*abs(exact-approx)/abs(exact)), (exact, approx));
Pdots := seq([Dots[j][1], P(Exact(Dots[j][1]), Dots[j][2])], j=1..11);

# Printing results and tables
# Make tables with a pencil, it saves time.
# ========================================
# To extract and print items 1,101,201,1001 from a list:
Dots1 := Dots[1], Dots[101], Dots[201], Dots[1001];

End of Maple Lab 4: Numerical Methods.