\[ y' = ky \]

**Growth- Decay**

\[ y = y_0 e^{kx} \]

*Solution: \( y = y_0 e^{kx} \)*

\( y_0 = \text{an arbitrary constant} \)

\[ y(0) \]

---

\[ \frac{du}{dt} = -h(u-u_0) \]

**Newton's Cooling**

\[ u = u_0 + A_0 e^{-ht} \]

*Solution: \( u = u_0 + A_0 e^{-ht} \)*

*Obtained by changing \( y = u-u_0 \)*

*To get \( y' = -hy \), Then apply \( A \) recipe above.*

---

\[ \frac{dp}{dt} = (a-bp)p \]

**Verhulst Logistic**

\[ p(t) = \frac{ap_0}{b + (a-bp_0)e^{-at}} \]

*Solution: \( p(t) = \frac{ap_0}{b + (a-bp_0)e^{-at}} \)*

*Where \( p_0 = p(0) = \text{initial population} \).*

*Obtained by changing \( \frac{y}{p} = \frac{a}{a-bp} \)*

*To get \( y' = ay \), Then apply \( A \) recipe above.*
1.2 - #1

1. Find a function \( y = f(x) \) satisfying the given differential equation and the prescribed initial condition.

\[
\frac{dy}{dx} = 2x + 1 \quad \text{Given}
\]

\[
y(x) = \int (2x + 1) \, dx
\]

\[
y(x) = x^2 + x + C
\]

\[
y(0) = 0 + 0 + C
\]

\[
0 + 0 + C = 3
\]

\[
C = 3
\]

\[
y(x) = x^2 + x + 3
\]

Check:

Back of Book
Apply the method of quadrature to solve
\[
\begin{aligned}
&y' = 2x + 1 \\
y(0) = 3
\end{aligned}
\]

\[\int y' \, dx = \int (2x + 1) \, dx\]

\[y = x^2 + x + c\]

\[3 = 0^2 + 0 + c\]

\[c = 3\]

\[y = x^2 + x + 3\]

**Check:**

LHS = \[y'\]

\[= (x^2 + x + 3)\]

\[= 2x + 1\]

\[= \text{RHS}\]

\[y(0) = 0^2 + 0 + 3\]

\[= 3\]

\[y = x^2 + x + 2\]

**Given DE**

Integrate across both sides on \(x\).

Fund. Thm. of calculus applied; \(c = \text{constant}\)

use \(y = 3\) at \(x = 0\)

**Candidate Solution found.**

LHS = left hand side

as \(y' = 2x + 1\), RHS = right hand side.

DE verified

Initial condition \(y(0) = 3\)

is verified.

**Solution.**
PROBLEM 2 pg. 17 #2

Find a function $y = f(x)$ which satisfies the given differential equation $\frac{dy}{dx} = (x-2)^2$ and initial condition $y(2) = 1$.

$y(x) = (x-2)^2$

$y'(x) \, dx = (x-2)^2 \, dx$

$\int y'(x) \, dx = \int (x-2)^2 \, dx$

$y(x) = \frac{(x-2)^3}{3} + C$

$1 = \frac{(2-2)^3 + C}{3}$

$C = 1$

$y(x) = \frac{(x-2)^3}{3} + 1$

Check:

$LHS = y'(x)$

$= \left[ \frac{(x-2)^3}{3} + 1 \right]'$

$= (x-2)^2 + 0$

$= RHS$

$LHS = y(2)$

$= \left[ \frac{(x-2)^3}{3} + 1 \right]_{x=2}$

$= 0 + 1$

$= RHS$

Checks with initial differential equation

Checks with initial condition $y(2) = 1$. 
Find a function \( y = y(x) \) which satisfies the differential equation \( \frac{dy}{dx} = (x-2)^2 \) and initial condition \( y(2) = 1 \).

\[
\begin{align*}
y'(x) &= (x-2)^2 \\
y'(x) \, dx &= (x-2)^2 \, dx \\
\int y'(x) \, dx &= \int (x-2)^2 \, dx \\
y(x) &= \frac{(x-2)^3}{3} + C \\
1 &= \frac{(2-2)^3}{3} + C \\
C &= 1 \\
y(x) &= \frac{(x-2)^3}{3} + 1
\end{align*}
\]

Given

Apply the method of quadrature

\[
\begin{align*}
\text{Use } y(2) = 1 \text{ to find } C
\end{align*}
\]

Candidate solution

Check:

LHS = \( y'(x) \)
\[
\begin{align*}
&= \left[ \frac{(x-2)^3}{3} + 1 \right]' \\
&= (x-2)^2 + 0 \\
&= \text{RHS}
\end{align*}
\]

LHS = \( y(2) \)
\[
\begin{align*}
&= \left[ \frac{(x-2)^3}{3} + 1 \right] \bigg|_{x=2} \\
&= 0 + 1 \\
&= \text{RHS}
\end{align*}
\]

DE verified

LHS side of IC

verified \( y(2) = 1 \)