Linear Transformation

A linear transformation is a function T defined on a vector space V with range in a vector space W satisfying the rules

- (a) $T(v_1 + v_2) = T(v_1) + T(v_2)$
- (b) $T(kv_1) = kT(v_1)$.

Theorem 1 (Matrix of T)

Assume $V = \mathbb{R}^n$ and $W = \mathbb{R}^m$. Then T is represented as a matrix multiply

$$T(\mathbf{x}) = A\mathbf{x}$$

where A is the $n \times m$ matrix whose columns are given in terms of the identity matrix I and function T by the formula

$$\operatorname{col}(A,j) = T(\operatorname{col}(I,j)), \quad j = 1, \dots, n.$$

Definition: A basis of a vector space V is a set of vectors v_1, \ldots, v_n such that every vector v in V can be uniquely written as a linear combination of v_1, \ldots, v_n . Briefly, the vectors *span* V and are *independent*.

Theorem 2 (Representation of T)

Every basis $\{v_1, \dots, v_n\}$ of V gives a relation

$$T\left(\sum_{j=1}^n c_j \mathrm{v}_j
ight) = \sum_{j=1}^n c_j \mathrm{w}_j, \quad ext{where} \quad \mathrm{w}_j = T(\mathrm{v}_j).$$