Linear Algebra 2270-2

Due in Week 6

The sixth week continues the work from chapter 3. Here's the list of problems, followed by problem notes and a few answers.

Section 3.3. Exercises 1, 12, 21

Section 3.4. Exercises 1, 3, 4, 6, 13, 16, 33

Problem Notes

Issues for Strang's problems will be communicated here. If there is a difficulty or impasse, then please send email, call 581-6879, or visit JWB 113.

Some Answers

3.3. Exercises 1, 21 have a textbook answer.

3.3-12. Invertible r by r submatrices use pivot rows and columns $S = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}$ and $S = [1]$ and $S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

3.4. Exercises 4, 6, 13, 16 have a textbook answer.

3.4-1. Row reduce the augmented matrix to upper triangular form

$$\left(\begin{array}{cccc|c} 2 & 4 & 6 & 4 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 0 & 0 & 0 & b_3 + b_2 - 2b_1 \end{array}\right)$$

Then $Ax = b$ has a solution when the last row is all zeros. This is the plane given by the equation $b_3 + b_2 - 2b_1 = 0$. The nullspace is obtained by solving $Ax = 0$, which is a step away by back-substitution. The answer is $\vec{x}_{\text{nullspace}} = c_1\vec{s}_1 + c_2\vec{s}_2$ where $\vec{s}_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{s}_2 = \begin{pmatrix} 2 \\ -2 \\ 0 \\ 1 \end{pmatrix}$. Then the complete solution is $c_1\vec{s}_1 + c_2\vec{s}_2 + \begin{pmatrix} b_1 \\ b_2 - b_1 \\ 0 \end{pmatrix}$, subject to the restraint $b_3 + b_2 - 2b_1 = 0$ (b_1, b_2 unrestrained). Choosing $b_1 = 4$ and $b_2 = 3$ with $c_1 = c_2 = 0$ gives particular solution $\vec{x}_{\text{particular}} = \begin{pmatrix} 4 \\ -1 \\ 0 \\ 0 \end{pmatrix}$.

3.4-3. $\vec{x}_{\text{complete}} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$. The matrix is singular but the equations are still solvable; b is in the column space. Our particular solution has free variable $y = 0$.

3.4-33. If the complete solution to $Ax = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix}$ then $A = \begin{pmatrix} 1 & 0 \\ 3 & 0 \end{pmatrix}$.