Linear Algebra 2270-2
Due in Week 12

The twelfth week continues eigenanalysis chapter 6. Here's the list of problems, followed by problem notes and answers.

Section 6.3. Exercises 4, 10, 18, 19

Section 6.4. Exercises 5, 7, 11, 14, 21, 23

Section 6.5. Exercises 3, 8, 10, 23, 24, 35

Problem Notes

Issues for Strang's problems will be communicated here. For help, please send email, call 581-6879, or visit JWB 113.

Some Answers

6.3. Exercise 4 has a textbook answer.

6.3-10. $A = \begin{pmatrix} 0 & 1 \\ 4 & 5 \end{pmatrix}$. $\lambda^2 - 5\lambda - 4 = 0$ is the characteristic equation of A with roots $\frac{5}{2} \pm \frac{1}{2}\sqrt{41}$. Check the characteristic equation by substitution of $y = e^{\lambda x}$ into the differential equation $y'' - 5y' - 4y = 0$.

6.3-18. Differentiate the matrix series for e^{At} as though A was a scalar to get the calculus answer $A + A^2 t + A^3 t^2/2 + \cdots$ which is exactly A times the infinite series for e^{At}.

6.3-19. $e^{Bt} = I + Bt$ (because B^2, B^3, ...are all the zero matrix). Then $e^{Bt} = \begin{pmatrix} 1 & -4t \\ 0 & 1 \end{pmatrix}$. Check $\frac{d}{dt}e^{Bt} = \begin{pmatrix} 0 & -4 \\ 0 & 0 \end{pmatrix}$ and $Be^{Bt} = B(I + Bt) = B + B^2 t = B + \text{zero matrix} = \begin{pmatrix} 0 & -4 \\ 0 & 0 \end{pmatrix}$.

6.4. Exercises 5, 11, 14, 21, 23 have textbook answers.

6.4-7. (a) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ has eigenvalues -1 and 3. (b) Each pivot has the same signs as the λs (c) trace $= \lambda_1 + \lambda_2 = 2$, so A cannot have two negative eigenvalues.

6.5. Exercises 3, 8, 10, 24 have textbook answers.

6.5-23. $x^2/a^2 + y^2/b^2$ is $x^T Ax$ when $A = \mathbf{diag}(1/a^2, 1/b^2)$. Then $\lambda_1 = 1/a^2$ and $\lambda_2 = 1/b^2$ so $a = 1/\sqrt{\lambda_1}$ and $b = 1/\sqrt{\lambda_2}$. The ellipse $9x^2 + 16y^2 = 1$ has axes with half-lengths $a = 1/3$ and $b = 1/4$. The points $(1/3, 0)$ and $(0, 1/4)$ are at the ends of the axes.

6.5-35. Put parentheses in $x^T A^T CAx$ to get $(Ax)^T C(Ax)$. Since C is assumed positive definite, this energy can drop to zero only when $Ax = 0$. Since A is assumed to have independent columns, then $Ax = 0$ only happens when $x = 0$. Thus $A^T CA$ has positive energy and it is positive definite.

Strang: My textbooks *Computational Science and Engineering* and *Introduction to Applied Mathematics* start with many examples of $A^T CA$ in a wide range of applications. I believe this is a unifying concept from linear algebra.