Linear Algebra 2270-2

Due in Week 10

The tenth week finishes determinants chapter 5. Extra problems are assigned from Fourier Series section 8.5, the background for maple lab 4. Here's the list of problems, problem notes and answers.

Problem week10-1. The first three Legendre polynomials are 1, x, and $x^2 - 1$. Choose c so that the fourth polynomial $x^3 - cx$ is orthogonal to the first three. All integrals go from -1 to 1. See Exercise 8.5-4.

Problem week10-2. Graph the square wave. Then graph by hand the sum of two sine terms in its series, or graph by machine the sum of 2, 3, and 10 terms. The famous Gibbs phenomenon is the oscillation that overshoots the jump (this doesn't die down with more terms). See Exercise 8.5-7.

Section 5.3. Exercises 2, 4, 6, 9, 17, 28, 33, 36

Problem Notes

Issues for Strang's problems will be communicated here. For help, please send email, call 581-6879, or visit JWB 113.

Some Answers

Problem week10-1. Answer: $c = 3/5$. Use $\int_{-T}^{T}(\text{odd function})dx = 0$ in the report.

Problem week10-2. The -1, 1 odd square wave is $f(x) = x/|x|$ for $0 < |x| < \pi$. Its Fourier series in equation (8), Section 8.5 of Strang's book, is $4/i$ times $[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x \cdots]$. The sum of the first N terms has an interesting shape, close to the square wave except where the wave jumps between -1 and 1. At those jumps, the Fourier sum spikes the wrong way to ± 1.09 (the Gibbs phenomenon) before it takes the jump with the true values of $f(x)$.

This happens for the Fourier sums of all functions with jumps. It makes shock waves hard to compute. You can see it clearly in a graph of the sum of 10 terms.

5.3. Exercises 2, 4, 6, 9, 17 have textbook answers.

5.3-28.Let $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$ and $\vec{u} = (x, y, z)$. Then

$$J = \det[\vec{u}_\rho|\vec{u}_\phi|\vec{u}_\theta] = \begin{pmatrix} \sin\phi\,\cos\theta & \rho\cos\phi\,\cos\theta & -\rho\sin\phi\,\sin\theta \\ \sin\phi\,\sin\theta & \rho\cos\phi\,\sin\theta & \rho\sin\phi\,\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{pmatrix}$$

Expand by cofactors along column 1, then simplify with trig identities, obtaining the answer $= \rho^2\sin\phi$.

5.3-33. Find the components of $\vec{v} \times \vec{w}$ from determinant expansion $\det\begin{pmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}$. These are exactly the cofactors along row one of the determinant $\det\begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}$. Then the determinant equals its cofactor expansion = dot product of $\vec{u}$ and $\vec{v} \times \vec{w}$.

5.3-36: If $(x, y, z), (1, 1, 0), (1, 2, 1)$ are in a plane, then the volume formed from the parallelepiped using these three vectors as edges must be zero. The volume is the determinant of the matrix whose rows are $(x, y, z), (1, 1, 0), (1, 2, 1)$. This volume is $x - y + z = 0$. The **box** with those edges is flattened to zero height.