MATH 2270-003 — Exam 2 — Fall 2010

NAME : ____________________

You may not use a calculator. Your solutions must include enough justification that another person could understand and be convinced by your argument.

There are extra blank pages at the end of the booklet. If you need more room to work a problem please note the page number where your work continues.

QUESTION	VALUE	SCORE
1	10	
2	15	
3	15	
4	15	
5	5	
6	5	
7	15	
8	5	
9	10	
10	15	
TOTAL	110	

1. (10 points) V is the span of the given vectors in $\mathbb{R}^4$. Find orthonormal vectors whose span is V.

$$\bar{v}_1 = \begin{pmatrix} 0 \\ -1 \\ 2 \\ 0 \end{pmatrix}, \bar{v}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 3\sqrt{2} \end{pmatrix}$$

2. (15 points) For the subspace V in the previous problem, give the matrix that projects $\mathbb{R}^4$ to V and the matrix that projects $\mathbb{R}^4$ to $V^\perp$.

3. (15 points) Find the least squares best fit line for the points $(0, 1)$, $(1, 2)$, $(2, 3)$, $(4, 4)$.

4. (15 points) For the following matrix, find the eigenvalues and the maximum number of linearly independent eigenvectors. Find this many linearly independent eigenvectors.

$$A = \begin{pmatrix} 4 & 1 \\ -5 & -2 \end{pmatrix}$$

5. (5 points) For the following matrix, find the eigenvalues and the maximum number of linearly independent eigenvectors. Find this many linearly independent eigenvectors.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

6. (5 points) Find the determinant of the following matrix:

$$\begin{pmatrix} 3 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

7. (15 points) Describe the orbits of the discrete linear dynamical system $\bar{v}_{i+1} = A\bar{v}_i$ for the matrix

$$A = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$$

8. (5 points) Suppose V is a 4 dimensional subspace of $\mathbb{R}^9$. Let P_V be the matrix that projects $\mathbb{R}^9$ onto the subspace V. What are the dimensions and rank of the matrix P_V?

9. (10 points) Suppose A is a 3×3 matrix whose entries all have absolute value less than or equal to 2. Find such a matrix that has $Det(A) \geq 30$. Is it possible to find such a matrix with $Det(A) \geq 50$? Find one or explain why it is impossible.

10. (15 points)

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

Find a matrix C such that CAC^{-1} is diagonal.

No new questions beyond this point.