MATH 2270-003   Exam 1   Fall 2010

NAME (please print): _____

You may not use a calculator. Your solutions must include enough justification that another person could understand and be convinced by your argument.

There are extra blank pages at the end of the booklet. If you need more room to work a problem please note the page number where your work continues.

1. (5 points) Is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ a unit vector? Explain.

2. (5 points) The matrix A is given below. What is the elimination matrix that eliminates the entries below the second pivot.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

3. (5 points) E and F are given below. What is the inverse of EF?

$$E = \begin{pmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$F = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

4. (5 points) Describe in words the effect of multiplying EA where E is the matrix in the previous problem and A is a 3×5 matrix.

5. (15 points) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Let $B = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$. Let $C = \begin{pmatrix} 1 & 7 \\ 1 & 2 \end{pmatrix}$. Calculate the following:

$$AB$$

$$BA$$

$$C^2$$

$$A^T \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$A(2B - C)$$

6. (20 points) Prove or disprove that the following sets of vectors form a basis for the indicated vector space:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \text{ in } \mathbb{R}^2$$

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \text{ in } \mathbb{R}^3$$

7. (25 points) Find all solutions to the equation $A\bar{x} = \bar{b}$ for

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 6 \\ 2 & 4 & 8 & 12 \end{pmatrix}$$

$$\bar{b} = \begin{pmatrix} 4 \\ 5 \\ 10 \end{pmatrix}$$

8. (15 points) Find all solutions to the equation $A\bar{x} = \bar{b}$ for

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{pmatrix}$$

$$\bar{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

9. (15 points) Consider a system of 3 linear equations in five variables. How many solutions can there be? Explain the possibilities.

No new questions beyond this point.