MATH 2270-2 Sample Exam 1 Spring 2012

1. (5 points) State the *Three Possibilities* for a linear system $A\vec{x} = \vec{b}$.

2. (5 points) Completely describe each operation in the basic Toolkit for solving a linear system (combo, swap, mult).

3. (5 points) Can the following system have no solution for some choice of b_1, b_2, b_3?

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= b_1 \\ 4x_1 + 2x_2 + 3x_3 \phantom{{}+x_4} &= b_2 \\ x_3 + x_4 &= b_3 \end{aligned}$$

4. (5 points) True or false? Explain. If A and B are $n \times n$ invertible, then $(AB)^{-1} = A^{-1}B^{-1}$.

5. (5 points) True or false? Explain. If square matrices A and B satisfy $AB = I$, then $A\vec{x} = \vec{b}$ has a unique solution $\vec{x}$ for each vector $\vec{b}$.

6. (5 points) Give an example of a 3×2 matrix A and a frame sequence with three or more frames, starting at A, which proves that the invented system $A\vec{x} = \vec{0}$ has a unique solution.

7. (5 points) Give an example of a 4×3 matrix A and a frame sequence with three or more frames, starting at A, which proves that the invented system $A\vec{x} = \vec{0}$ has infinitely many solutions.

8. (5 points) Let A be a 3×4 matrix. What is the elimination matrix that replaces Row 2 of A with Row 2 minus Row 1 and replaces Row 3 of A by Row 3 minus 2 times Row 1?

9. (5 points) Let A be a 3×3 matrix. Let

$$F = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Assume F is obtained from A by the following sequential row operations: (1) Swap rows 2 and 3; (2) Add -2 times row 2 to row 3; (3) Add 3 times row 1 to row 2; (4) Multiply row 2 by -3. Find A.

10. (5 points) What is the inverse of the following matrix?

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

11. (5 points) Describe in words the effect of multiplying

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$

on the left of a 4×5 matrix A to get EA.

12. (20 points) Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b-c & a \\ 1 & c & -a \\ 2 & b & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

(a). Determine those values of a, b and c such that the system has a unique solution.

(b). Determine those values of a, b and c such that the system has no solution.

(c). Determine those values of a, b and c such that the system has infinitely many solutions.

To save time, don't attempt to solve the equations or to produce a complete frame sequence.

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macro(combo=linalg[addrow]);macro(mult=linalg[mulrow]);
macro(swap=linalg[swaprow]);
A:=(a,b,c)->Matrix([[1,b-c,a,-a],[1,c,-a,a],[2,b,a,a]]);
A1:=combo(A(a,b,c),1,2,-1);
A2:=combo(A1,1,3,-2);
A3:=combo(A2,2,3,-1);
A4:=combo(A3,3,2,2);
A5:=combo(A4,3,1,-1);
A5 := Matrix([[1,b-c,0,-2*a],[0,-b+2*c,0,4*a],[0,0,a,a]]);
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13. (15 points) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Let $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Let $C = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Calculate the following:

$$A(BC), \quad (AB)C, \quad C^2, \quad 2A + B - C, \quad A(B - C)$$

14. (20 points) Classify the following sets of vectors for (1) Independence or (2) Dependence. For each set of vectors, report whether or not they form a **basis** for the indicated vector space.

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \text{ in } \mathbb{R}^2$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \text{ in } \mathbb{R}^3$$

15. (20 points) Find all solutions to the equation $A\bar{x} = \bar{b}$ for

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$$

$$\bar{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

16. (20 points) Prove that a system of 4 linear equations in 5 unknowns has either no solution or infinitely many solutions.

17. (20 points) Let matrix A have 201 rows and 201 columns. All entries of A are zero off the diagonal, except for the number -7 in row 107, column 35. The diagonal entries of A are all one. Describe the inverse of A, in words.

End of the sample exam questions.