MATH 2270-2 Exam 1 Spring 2012

1. (10 points) Give a counter example or explain why it is true. If A and B are $n \times n$ invertible, and C^T denotes the transpose of a matrix C, then $(AB^{-1})^T = (B^T)^{-1}A^T$.

2. (10 points) Give a counter example or explain why it is true. If square matrices A and B satisfy $AB = I$, then the transposes satisfy $A^T B^T = I$.

3. (10 points) Let A be a 3×4 matrix. Find the elimination matrix E which under left multiplication against A performs both (1) and (2) with one matrix multiply.

(1) Replace Row 2 of A with Row 2 minus Row 3.

(2) Replace Row 3 of A by Row 3 minus 4 times Row 1.

4. (30 points) Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b & c \\ 1 & c & -a \\ 2 & b+c & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

Use techniques learned in this course to briefly explain the following facts. Only write what is needed to justify a statement.

(a). The system has a unique solution for $(c-b)(2a-c) \neq 0$.

(b). The system has no solution if $c = 2a$ and $a \neq 0$ (don't explain the other possibilities).

(c). The system has infinitely many solutions if $a = b = c = 0$ (don't explain the other possibilities).

Continued

Definition. *Vectors $\vec{v}_1, \ldots, \vec{v}_k$ are called* **independent** *provided solving the equation $c_1\vec{v}_1 + \cdots + c_k\vec{v}_k = \vec{0}$ for constants $c_1, \ldots, c_k$ has the unique solution $c_1 = \cdots = c_k = 0$. Otherwise the vectors are called* **dependent**.

5. (20 points) Classify the following sets of vectors as Independent or Dependent, using the Pivot Theorem or the definition of independence (above).

$$\text{Set 1:} \quad \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\text{Set 2:} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

6. (20 points) Find the vector general solution $\vec{x}$ to the equation $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

End Exam 1.