

Digital Photographs and Matrices

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Digital Photographs

A digital camera stores image sensor data as a matrix A of numbers corresponding to the color and intensity of tiny sensor sites called **pixels** or **dots**. The pixel position in the print is given by row and column location in the matrix A .

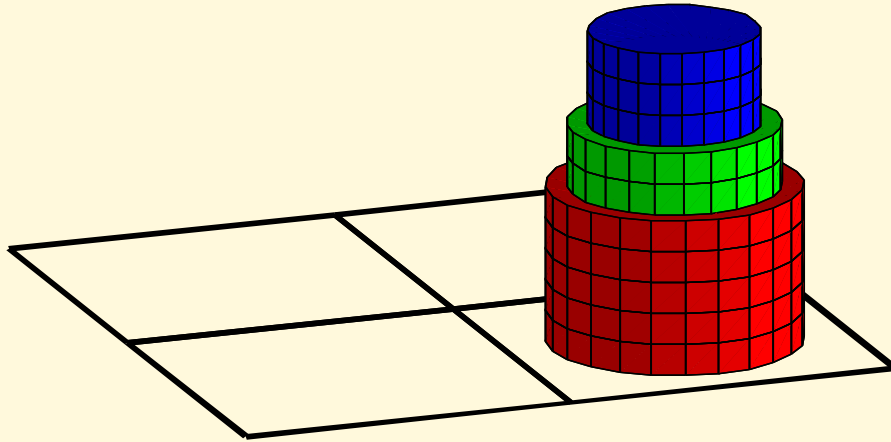


Figure 1. Checkerboard visualization.

Illustrated is a stack of checkers, representing one photodiode site on an image sensor inside a digital camera. There are 5 red, 2 green and 3 blue checkers stacked on one square. The checkers represent the number of electrons knocked loose by photons falling on each RGB-filtered site.

Color Model for 24-bit

In 24-bit color, a pixel could be represented in matrix A by a coded integer

$$a = r + (2^8)g + (2^{16})b.$$

Symbols r , g , b are integers between 0 and 255 which represent the intensity of colors red, green and blue, respectively. For example, $r = g = b = 0$ is the color **black** while $r = g = b = 255$ is the color **white**. Grander schemes exist, e.g., 32-bit and 128-bit color.^a

^a A typical beginner's digital camera makes low resolution color photos using 24-bit color. The photo is constructed of 240 rows of dots with 320 dots per row. The associated storage matrix A is of size 240×320 . The identical small format is used for video clips at up to 30 frames per second in video-capable digital cameras.

The storage format **BMP** stores data as bytes, in groups of three b , g , r , starting at the lower left corner of the photo. Therefore, 240×320 photos have 230,400 data bytes. The storage format **JPEG** reduces file size by compression and quality loss.

Visualization of Matrix Addition

Matrix addition can be visualized through matrices representing color separations, a technique invented by James Clerk Maxwell.

When three monochrome transparencies of colors red, green and blue (RGB) are projected simultaneously by a projector, the colors add to make a full color screen projection.

The three transparencies can be associated with matrices \mathbf{R} , \mathbf{G} , \mathbf{B} which contain pixel data for the monochrome images. Then the projected image is associated with the matrix sum $\mathbf{R} + \mathbf{G} + \mathbf{B}$.

Visualization of Matrix Scalar Multiplication

Scalar multiplication of matrices has a similar visualization.

The pixel information in a monochrome image (red, green or blue) is coded for intensity.

The associated matrix \mathbf{A} of pixel data when multiplied by a scalar k gives a new matrix $k\mathbf{A}$ of pixel data with the intensity of each pixel adjusted by factor k .

The photographic effect is to adjust the range of intensities. In the checkerboard visualization of an image sensor, factor k increases or decreases the checker stack height at each square.

Color Separation Illustration

Consider the coded matrix

$$\mathbf{X} = \begin{pmatrix} 514 & 3 \\ 131843 & 197125 \end{pmatrix}.$$

We will determine the monochromatic pixel data \mathbf{R} , \mathbf{G} , \mathbf{B} in the equation $\mathbf{X} = \mathbf{R} + 2^8\mathbf{G} + 2^{16}\mathbf{B}$.

First we decode the scalar equation $x = r + 2^8g + 2^{16}b$ by these algebraic steps, which use the modulus function $\text{mod}(x, m)$, defined to be the remainder after division of x by m . We assume r, g, b are integers in the range 0 to 255.

$$y = \text{mod}(x, 2^{16})$$

The remainder should be $y = r + 2^8g$.

$$r = \text{mod}(y, 2^8)$$

Because $y = r + 2^8g$, the remainder equals r .

$$g = (y - r)/2^8$$

Divide $y - r = 2^8g$ by 2^8 to obtain g .

$$b = (x - y)/2^{16}$$

Because $x - y = x - r - 2^8g$ has remainder b .

$$r + 2^8g + 2^{16}b$$

Answer check. This should equal x .

Decoding with a Computer Algebra System

Computer algebra systems can provide an answer for matrices R , G , B by duplicating the scalar steps. Below is a maple implementation that gives the answers

$$R = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}, G = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 2 & 3 \end{pmatrix}.$$

```
with(LinearAlgebra:-Modular):  
X:=Matrix([[514,3],[131843,197125]]);  
Y:=Mod(2^16,X,integer); # y=mod(x,65536)  
R:=Mod(2^8,Y,integer); # r=mod(y,256)  
G:=(Y-R)/2^8; # g=(y-r)/256  
B:=(X-Y)/2^16; # b=(x-y)/65536  
X-(R+G*2^8+B*2^16); # answer check
```

The Checkerboard Visualization

The result can be visualized through a checkerboard of 4 squares. The second square has 5 red, 2 green and 3 blue checkers stacked, representing the color $x = (5) + 2^8(2) + 2^{16}(3)$ - see Figure 1. A matrix of size $m \times n$ is visualized as a checkerboard with mn squares, each square stacked with red, green and blue checkers.

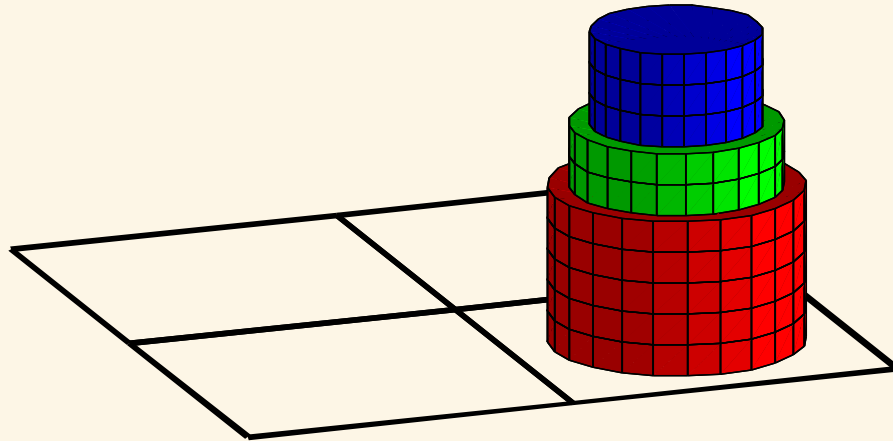


Figure 2. Checkerboard visualization.

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