Name KEY

Differential Equations and Linear Algebra 2250

Midterm Exam 2

Version 2a, Thu 29 March 2012

Scores
4.
5.

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

4. **(Determinants)** Do all parts.

(a) [20%] State four different determinant rules for $n \times n$ matrices.

(b) [20%] Assume given 3×3 matrices A, B. Suppose $AB = E_3E_2E_1A$ and E_1, E_2, E_3 are elementary matrices representing respectively a swap, a combination, and a multiply by $-1/3$. Assume $\det(A) = 13$. Find $\det(2B)$.

(c) [20%] Determine all values of x for which B^{-1} fails to exist, where B equals the transpose of the matrix $\begin{pmatrix} 2 & 0 & 5x & 0 \\ 3x & 0 & 10 & 0 \\ 1 & x-1 & 7 & 0 \\ x^4 & x^3 & x^2 & x \end{pmatrix}$.

(d) [40%] Apply the adjugate [adjoint] formula for the inverse to find the value of the entry in row 3, column 4 of A^{-1}, given A below. Other methods are not acceptable.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 2 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

(a) Triangular rule, swap rule, combo rule, multiply rule, cofactor rule, transpose rule, rules for zero determinant, product rule, sum rule

(b) $|2B| = 2^3|B|$ and $|A||B| = |E_3||E_2||E_1||A|$ by The product rule for determinants. Because $|E_3| = -1/3$, $|E_2| = 1$, $|E_1| = -1$, Then $|B| = \frac{1}{3}$, So $|2B| = 2^3 \frac{1}{3} = 8/3$.

(c) B^{-1} fails to exist when $|B| = 0$. But $|B| = |B^T| = x\begin{vmatrix} 2 & 0 & 5x \\ 3x & 0 & 10 \\ 1 & x-1 & 7 \end{vmatrix} = -x(x-1)\begin{vmatrix} 2 & 5x \\ 3x & 10 \end{vmatrix}$ $= -x(x-1)(20 - 15x^2)$. So B^{-1} fails to exist for roots of $-x(x-1)(4-3x^2)5 = 0$, which is $x = 0, 1, \pm 2/\sqrt{3}$.

(d) entry in row 3, col 4 of $A^{-1} = \dfrac{\text{cofactor}(A,4,3)}{|A|} = \dfrac{(-1)^{4+3}\text{minor}(A,4,3)}{|A|}$.

$|A| = 1\cdot\begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{vmatrix} - 1\cdot\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & -2 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{vmatrix}$ by cofactor expansion on col(A, 3) and The sum rule for determinants

$|A| = (-1)(-1)\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} + (-2)(+1)\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1$

$\text{minor}(A,4,3) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 2 & 2 \end{vmatrix} = \underbrace{\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 1 \end{vmatrix}}_{\text{2 combos}} = 1 \quad \Rightarrow \quad \text{entry} = \dfrac{(-1)^{4+3}(1)}{(-1)} = \boxed{1}$

Use this page to start your solution. Attach extra pages as needed.

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5. **(Linear Differential Equations)** Do all parts.

(a) [20%] Solve for the general solution of $12y'' + 7y' + y = 0$.

(b) [40%] The characteristic equation is $r^2(2r-3)^2(r^2-2r+5) = 0$. Find the general solution y of the linear homogeneous constant-coefficient differential equation.

(c) [20%] A third order linear homogeneous differential equation with constant coefficients has two particular solutions $2e^{3x} + 4\sin 2x$ and e^{3x}. What are the roots of the characteristic equation?

(d) [20%] **Circle the functions** which can be a solution of a linear homogeneous differential equation with constant coefficients. For example, you would circle $\cos^2 x$ because $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$ is a linear combination of the two solutions 1 and $\cos(2x)$ of a third order equation whose characteristic equation has roots 0, $2i$, $-2i$.

$e^{\ln|2x|}$ $\quad e^{x^2}$ $\quad e + x$ $\quad \cos(\ln|x|)$ $\quad \tan x$

$\cos(x\ln|3.7125|)$ $\quad x^{-1}e^{-x}\sin(\pi x)$ $\quad \cosh x$ $\quad \sin^2 x$ $\quad \cos(x^2)$

(a) $12r^2 + 7r + 1 = (4r+1)(3r+1) \Rightarrow r = -1/4, -1/3 \Rightarrow$ atoms $= e^{-x/4}, e^{-x/3}$
y = linear combination of the atoms.

(b) $r^2(2r-3)^2((r-1)^2+4) = 0$ has roots $0, 0, 3/2, 3/2, 1 \pm 2i$, atoms $=$ $1, x, e^{3x/2}, xe^{3x/2}, e^x\cos 2x, e^x\sin 2x$. Then y = linear combination of the atoms.

(c) The DE must have atoms e^{3x}, $\cos 2x$, $\sin 2x$ So the roots are $r = 3, \pm 2i$.

(d) $e^{\ln|2x|} = |2x|$ not an atom; e^{x^2} not an atom; $e+x$ = l.c. of atoms; $\cos(\ln|x|)$ not an atom; $\tan x$ not an atom; $\cos(bx)$ is an atom for $b = \ln|3.7125|$; x^{-1} cannot be a factor of an atom; $\cosh(x) = \frac{e^x + e^{-x}}{2}$ is a linear combination of atoms; $\sin^2 x = \frac{1-\cos 2x}{2}$ is a l.c. of atoms; $\cos(x^2)$ is not an atom.

Use this page to start your solution. Attach extra pages as needed.