Name ________________________ **2270-3 Midterm 3 F2008**

Introduction to Linear Algebra 2270-3
Midterm Exam 3 Fall 2008
Exam Date: Wednesday, 3 December 2008

Instructions. The exam is 50 minutes. Calculators are not allowed. Books and notes are not allowed.

1. (Orthogonality, Gram-Schmidt) Complete enough to make 100%.

(1a) [40%] Find the orthogonal projection of $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ onto $V = \mathbf{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\right\}$.

(1b) [40%] Find the QR-factorization of $A = \begin{pmatrix} 1 & -2 \\ 0 & 7 \\ 1 & 2 \end{pmatrix}$.

(1c) [20%] Prove that any power A^n of an orthogonal matrix A is again orthogonal.

(1d) [20%] Prove that $\mathbf{ker}(A^T) = \mathbf{im}(A)^\perp$.

(1e) [20%] Prove that the Gram-Schmidt vector $\mathbf{u}_3$ is not in the span of $\mathbf{u}_1$, $\mathbf{u}_2$. These vectors are constructed from independent vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ by the Gram-Schmidt formulas.

Please start your solutions on this page. Additional pages may be stapled to this one.

2. (Determinants) Complete enough to make 100%.

(2a) [50%] Evaluate $\det(E - I)$, where E is an elementary swap matrix and I is the identity matrix, for matrices of size $n \times n$, $n = 2, 3, \ldots, 10$. Supply statements of theorems which were used to evaluate the determinant. Missing details subtract credit.

(2b) [25%] Find A^{-1} by the classical adjoint method:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}.$$

(2c) [25%] Let 3×3 matrix A be invertible and assume $\mathbf{rref}(A) = E_4 E_3 E_2 E_1 A$. The elementary matrices E_1, E_2, E_3, E_4 represent `combo(1,3,-15)`, `swap(1,4)`, `mult(2,-1/4)`, `combo(1,2,-2)`, respectively. Let B be a 3×3 orthogonal matrix. Find all possible values of $\det(A^3 B^{-1} (B^T)^2)$, where B^T is the transpose of B.

(2d) [25%] Let B be the matrix given below, where $\boxed{?}$ means the value of the entry does not affect the answer to this problem. The second matrix C is the adjugate (or adjoint) of B. Find the value of $\det(BC)$.

$$B = \begin{pmatrix} -2 & ? & -2 & 3 \\ ? & ? & 1 & -2 \\ ? & 1 & 2 & ? \\ ? & 0 & ? & ? \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 0 & 1 \\ -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 2 \end{pmatrix}$$

Please start your solutions on this page. Additional pages may be stapled to this one.