Name ________________________ **2270-3**

Introduction to Linear Algebra 2270-3

Midterm Exam 2 Fall 2008

Draft of 8 November 2008

Take-home problems due November 17

1. (Matrices, bases and independence)

(a) Prove that the column positions of leading ones in $\mathbf{rref}(A)$ identify columns of A which form a basis for $\mathbf{im}(A)$.

(b) Find a basis for the image of any invertible $n \times n$ matrix.

(c) Let T be the linear transformation on $\mathcal{R}^3$ defined by mapping the columns of the identity respectively into three independent vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$. Define $\mathbf{u}_1 = \mathbf{v}_1 + 2\mathbf{v}_3$, $\mathbf{u}_2 = \mathbf{v}_1 + 3\mathbf{v}_2$, $\mathbf{u}_3 = \mathbf{v}_2 + 4\mathbf{v}_3$. Verify that $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a basis for $\mathcal{R}^3$ and report the $\mathcal{B}$-matrix of T (Otto Bretscher 3E, page 142).

Please staple this page to the front of your submitted exam problem 1.

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Take-Home Problems Due 17 November

2. (Kernel and similarity)

(a) Prove or disprove: $AB = I$ with A, B possibly non-square implies $\mathbf{ker}(A) = \{\mathbf{0}\}$.

(b) Prove or disprove: $\mathbf{ker}(\mathbf{rref}(BA)) = \mathbf{ker}(A)$, for all invertible matrices B.

(c) Prove or disprove: $\mathbf{im}(\mathbf{rref}(BA)) = \mathbf{im}(A)$, for all invertible matrices B.

(d) Prove or disprove: Similar matrices A and B satisfy $\mathbf{nullity}(A) = \mathbf{nullity}(B)$.

Please staple this page to the front of your submitted exam problem 2.

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3. (Independence and bases)

(a) Let A be an $n \times m$ matrix. Report a condition on A such that all possible finite sets of independent vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are mapped by A into independent vectors $A\mathbf{v}_1, \ldots, A\mathbf{v}_k$. Prove that any matrix A satisfying the condition maps independent sets into independent sets.

(b) Let V be the vector space of all polynomials $c_0 + c_1x + c_2x^2$ under function addition and scalar multiplication. Prove that $1 - x$, $2x + 1$, $(x - 1)^2$ form a basis of V.

Please staple this page to the front of your submitted exam problem 3.

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4. (Linear transformations)

(a) Let L be a line through the origin in $\mathcal{R}^3$ with unit direction $\mathbf{u}$. Let T be a reflection through L. Define T precisely. Compute and display its representation matrix A, i.e., the unique matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

(b) Let T be a linear transformation from $\mathcal{R}^n$ into $\mathcal{R}^m$. Given a basis $\mathbf{v}_1, \ldots, \mathbf{v}_n$ of $\mathcal{R}^n$, let A be the matrix whose columns are $T(\mathbf{v}_1), \ldots, T(\mathbf{v}_n)$. Prove that $T(\mathbf{x}) = A\mathbf{x}$.

(c) Consider the equations

$$\begin{aligned} I &= \frac{1}{3}(R + G + B) \\ L &= R - G \\ S &= B - \frac{1}{2}(R + G). \end{aligned}$$

On page 94 of Otto Bretscher 3E, these equations are discussed as representing the intensity I, long-wave signal L and short-wave signal S in terms of the amounts R, G, B of red, green and blue light. Submit all parts of problem 86, page 94.

In the last part 86d, let T be the eye-brain transformation with matrix M and let T_1 be the transformation in 86a, having matrix P. Otto wants $T_1 T$ to be the sunglass-eye-brain composite transformation of 86c. This explains why 86c and 86d are different questions. A class discussion will help to clarify the Bretscher statement of the problem.

Please staple this page to the front of your submitted exam problem 4.

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5. (Vector spaces)

(a) Show that the set of all 5×4 matrices A which have exactly one element equal to 1, and all other elements zero, form a basis for the vector space of all 5×4 matrices.

(b) Let W be the set of all functions defined on the real line, using the usual definitions of function addition and scalar multiplication. Let V be the set of all polynomials spanned by 1, x, x^2, x^3, x^4. Assume W is known to be a vector space. Prove that V is a subspace of W.

Please staple this page to the front of your submitted exam problem 5.