Name ___________________________ Class Time _____

Math 2270 Extra Credit Problems
Chapter 4
S2010

Due date: See the internet due date for 6.3, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem XC4.1-20. (Matrix space basis)

Find a basis and the dimension for the space of all 2×2 matrices $\begin{pmatrix} a & b \\ a+b & a-b \end{pmatrix}$.

Problem XC4.1-26. (Polynomial space basis)

Find a basis and the dimension for the space of all polynomials $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ satisfying the conditions $p(-1) = p(1)$, $\int_0^1 p(x)dx = p(0)$.

Problem XC4.1-38. (Commutator dimension)

Find the dimension of the space of all 3×3 matrices A that commute with a 3×3 diagonal matrix B.

Problem XC4.1-54. (Subspace dimension)

Let V be a vector space of dimension n and let S be a subspace of V. Prove that S has a basis of k elements and $k \le n$.

Problem XC4.2-13. (Linear transformations and isomorphisms)

Let $T(A) = PA - AP$ where $P = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$. Prove that T is a linear transformation on the set V of all 2×2 matrices A. Determine the kernel and image of T and report whether T is an isomorphism.

Problem XC4.2-20. (Kernel of a linear transformation)

Let $T(y) = 3y'' + 5y'$ be defined on the vector space V of all twice continuously differentiable functions $y(x)$ defined on $-\infty < x < \infty$ with range in some vector space W. Compute the kernel of T and therefore show that T is not an isomorphism. How should W be defined? Is there a problem with $W = V$?

Hint: Write $v = y'$ and break the equation $3y'' + 5y' = 0$ into $3v' + 5v = 0$ and $y' = v$. These are elementary first order differential equations. The general solution of $3v' + 5v = 0$ is $v = v_0 e^{-5x/3}$.

Problem XC4.2-34. (Sequence spaces)

Let $T(f)$ denote the sequence $f(0)$, $f'(0)$, ... of derivatives of f evaluated at 0. Let V be the space of all polynomials. Then the vector space W is the set of all sequences with only finitely many nonzero terms. Prove that W is a subspace of the vector space E of all real sequences equipped with termwise addition and scalar multiplication. Prove that T is linear and decide if it is an isomorphism.

Problem XC4.3-2. (Independence in matrix spaces)

Prove or disprove: a list of 2×2 matrices of length 5 or more is dependent.

Problem XC4.3-30. (Tangents)

Let $T(p) = p(0) + xp'(0)$ be defined on the polynomials $p(x) = a_0 + a_x + a_2 x^2$ of degree 2 or less, a vector space V. Prove that T is linear and compute the matrix of T relative to the basis 1, $x - 1$, $(x - 1)^2$.

Problem XC4.3-60. (Matrix of change of basis)

Let a plane P be given in $\mathcal{R}^3$ having two different bases $\{\mathbf{v}_1, \mathbf{v}_2\}$ and $\{\mathbf{w}_1, \mathbf{w}_2\}$. Give general formulas for the change of basis matrices V and W which change one basis into the other (in both possible ways). Then give a formula which relates V to $\mathbf{aug}(\mathbf{v}_1, \mathbf{v}_2)$ and $\mathbf{aug}(\mathbf{w}_1, \mathbf{w}_2)$.

Problem XC4.3-64. (Matrix of a linear transformation)

Let $T(A) = PA - AP$ where $P = \begin{pmatrix} 2 & 3 \\ 0 & 7 \end{pmatrix}$. Prove that T is a linear transformation on the set V of all 2×2 upper triangular matrices A. (a) Find the standard basis $\mathcal{B}$ of V. (b) Find the matrix of T relative to basis $\mathcal{B}$. (c) Determine the rank of T.

End of extra credit problems chapter 4.