Group 1

a. **im**($A^T B^T$) = **im**($(BA)^T$). Let $C = BA$, than **im**(C^T) = perp(**ker**(C)), which is true from the latter portion of the fundamental theorem of linear algebra.

b. An **MxN** matrix can be thought of a series of N vectors spanning some some space in R^N. Furthermore, any matrix that has an rref *I*, can become *I* by multiplying it by a series of elimentory matrices *E*, this works, so long as the coulomb count of *E*, matches the space of the matrix in R^N, the transformations are valid. Thus, this is analogous to finding the rref, and taking on coulomb.

c. True, If $AB = I$, than A is a series of elementary matrices *E*, such that it takes *B* to the identity. Thus, rref(*B*)=I, and thus, ker(*B*)={**0**}.

d. When some matrix *A* is multiplied by any other matrix, it's kernel can only expand, not contract. Next, the rank of A^T is the same as the rank of *A*, finaly, $A^T A$ should form a square matrix. Thus, the **ker**(*A*) = **{0}** impales that **ker**($A^{-1}A$) = **{0}**

e. False. Two matrices are similar if there is some way we can change the basis of one, into the other, or *A* and *B* and similar, if for some matrix *P*, $A=P^{-1}BA$. Consider the 1x1 matrix [1] and [2], they both have a kernel of 0, but they are not similar.

f. If some matrix *B* is invertible, than it can be obtained by multiplying together many eliminator matrices *E*. Finally, multiplying a matrix by an elementary matrix doesn't change it's kernel.

g. The **ker(rref(***A***))** = **ker(***A***)**, because finding the kernel of *A*, requires calculating the rref, and taking it on that. Finally, multiplying some matrix *A*, by only alimentary matrices *E*, will only change the basis that the matrix *A*, is using, meaning that the **ker(***A***)** is unchanged.

h. For all *n x n* matrix with rank of *n*, than that matrix has *n* linearly independent vectors, spanning a space containing *n* basis. Thus, if two matrices that are *n x n* exist, with *n* linearly independent vectors, they will be matrices of the same space.