Applied Linear Algebra 2270-2

Sample Midterm Exam 1

Thursday, 25 Feb 2010

Instructions: This in-class exam is designed for 50 minutes. No tables, notes, books or calculators allowed.

1. (Inverse of a matrix) Supply details for two of these:

a. If A and B are $n \times n$ invertible, then $(AB)^{-1} = B^{-1}A^{-1}$.

b. If possibly non-square matrices A and B satisfy $AB = I$, then $B\mathbf{x} = \mathbf{0}$ cannot have infinitely many solutions.

c. Give an example of a 4×3 system having a unique solution.

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2. (Elementary Matrices) Let A be a 3×3 matrix. Let

$$F = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Assume F is obtained from A by the following sequential row operations: (1) Swap rows 1 and 3; (2) Add -2 times row 2 to row 3; (3) Add 3 times row 1 to row 2; (4) Multiply row 2 by 5.

a. Write a matrix multiplication formula for F in terms of explicit elementary matrices and the matrix A. (80%)

b. Find A. (20%)

3. (RREF method)

Part I. State a theorem about homogeneous systems which concludes that the $m \times n$ system has at least one nonzero solution. Then supply a proof of the theorem. [20%]

Part II. Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b-c & a \\ 1 & c & -a \\ 2 & b & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

a. Determine those values of a, b and c such that the system has a unique solution. (40%)

b. Determine those values of a, b and c such that the system has no solution. (20%)

c. Determine those values of a, b and c such that the system has infinitely many solutions. (20%)

To save time, don't attempt to solve the equations or to produce a complete frame sequence.

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4. (Matrix algebra)
Do two of these:

a. Find all 2×2 matrices A such that $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} A = A \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + A$.

b. Let A be a 3×2 matrix and B a 2×3 matrix. Explain using matrix algebra and the three possibilities why the 3×3 matrix $C = AB$ cannot satisfy $\mathbf{rref}(C) = I$.

c. For 2×2 matrices A, B, prove that $(A + B)(A - B) = A^2 - B^2$ implies that A and B commute.

5. (Geometry and linear transformations)

Part I. Classify $T(\mathbf{x}) = A\mathbf{x}$ geometrically as scaling, projection onto line L, reflection in line L, pure rotation by angle θ, rotation composed with scaling, horizontal shear, vertical shear. [60%]

a. $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

b. $A = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$

c. $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

d. $A = \frac{1}{4}\begin{pmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

e. $A = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix}$

Part II. Give details. [40%]

f. Define reflection in a line L in $\mathcal{R}^3$.

g. Display the matrix A of a projection onto a line L in $\mathcal{R}^3$.

h. Define rotation clockwise by angle θ in $\mathcal{R}^2$.

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