

SOLVING a Linear System Snapshot Frame Sequence

Frame 1	Frame 2	Frame 3	
$\begin{cases} x+y+z=1 \\ x+y-z=3 \\ 2x+2y=4 \end{cases}$	$\begin{cases} x+y+z=1 \\ -2z=2 \\ 2x+2y=4 \end{cases}$	$\begin{cases} x+y+z=1 \\ -2z=2 \\ 0+0-2z=2 \end{cases}$	<u>3 possibilities</u> <ul style="list-style-type: none"> • No Sol • ∞-Many sols • Unique sol
Original System	Combo(1,2,-1)	Combo(1,3,-2)	<u>Toolkit of 3 Tools</u> <ul style="list-style-type: none"> • mult(t,c) • Swap(s,t) • Combo(s,t,c) <p style="font-size: small; margin-left: 20px;"> s = source t = target c = multiplier </p>
$A = \begin{array}{ccc c} x & y & z & \\ \hline 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 3 \\ 2 & 2 & 0 & 4 \end{array}$	$A_1 = \begin{array}{ccc c} x & y & z & \\ \hline 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & 2 \\ 2 & 2 & 0 & 4 \end{array}$	$A_2 = \begin{array}{ccc c} x & y & z & \\ \hline 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & -2 & 2 \end{array}$	
Orig sys, matrix form	Combo(1,2,-1)	Combo(1,3,-2)	

Object: Increase in each frame the count of lead vars, or else decrease the repeats of a var destined to become a lead var.

Frame 4	Frame 5	Frame 6	
$\begin{cases} x+y+z=1 \\ -2z=2 \\ 0=0 \end{cases}$	$\begin{cases} x+y+z=1 \\ z=-1 \\ 0=0 \end{cases}$	$\begin{cases} \boxed{x}+y=2 \\ \boxed{z}=-1 \\ 0=0 \end{cases}$	<u>Lead var</u> : has Coeff = 1, appears first left-to-right, appears exactly once in the system.
Combo(2,3,-1)	mult(2,-1/2)	Combo(2,1,-1)	<u>Free var</u> : all non-lead var. missing var is a free var.
$A_3 = \begin{array}{ccc c} x & y & z & \\ \hline 1 & 1 & 1 & -1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}$	$A_4 = \begin{array}{ccc c} x & y & z & \\ \hline 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}$	$A_5 = \begin{array}{ccc c} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}$	
Combo(2,3,-1)	mult(2,-1/2)	Combo(2,1,-1)	

Reduced echelon sys (frame 6): each nonzero equation has a lead var.

General Sol algorithm: assign invented symbols t_1, t_2, t_3, \dots to the free vars
 solve for lead vars, back-substitute. Write the var list in terms of symbols t_1, t_2, t_3, \dots

alg step 1
 $y = t_1$

alg step 2
 $\boxed{x} = 2 - y$
 $\boxed{z} = -1$

Gen Sol

$$\begin{cases} x = 2 - t_1 \\ y = t_1 \\ z = -1 \end{cases}$$

uns:
 $x = 2 - t_1$
 $y = t_1$
 $z = -1$